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STRESSES AND THRUSTS

A TEXT-BOOK FOR STUDENTS

BY

G. A. T. MIDDLETON

ASSOCIATE OF THE ROYAL INSTITUTE OF BRITISH ARCHITECTS AND
MEMBER OF THE SOCIETY OF ARCHITECTS
AUTHOR OF "THE DRAINAGE OF TOWN AND COUNTRY HOUSES, "SURVEYING AND
SURVEYING INSTRUMENTS," "ARCHITECTURAL PERSPECTIVE,' ETC.

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LONDON AND TONBRIDGE.

PREFACE TO THE THIRD EDITION.

A FIFTH edition of this volume (the third under its present title) being called for, the opportunity has been taken to still further enlarge and improve it. The chapters upon Rectangular Beams and upon Arches have been entirely remodelled and examples added, and to that upon Struts a worked example is now given, as well as some useful tables and a section upon Wooden Pillars, while the single example hitherto given of "Designing a Wrought-Iron Girder" has been replaced by two new ones dealing with "Designing a Steel Joist" and "Designing a Steel Plate Girder" respectively. A few minor alterations have likewise been made, and no trouble has been spared to make the book more useful than before to those to whom the correct determination of Stresses and Thrusts in ordinary constructional work is a matter of importance. A few examination papers have been added for the assistance of the many others who take up the subject for examination purposes.

G. A. T. MIDDLETON.

January, 1904.

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STRESSES AND THRUSTS.

CHAPTER I.

ELEMENTARY PRINCIPLES.

Before commencing the study of any subject demanding close logical reasoning, it is obviously necessary that the elementary principles involved should be thoroughly mastered, else the sound foundation is wanting upon which all else has to be erected.

As in all branches of applied statics, the principles involved in the consideration of Stresses and Thrusts are those of—

(*a*) Moments round a point.

(*b*) The parallelogram, triangle and polygon of forces; and

(*c*) The centre of gravity.

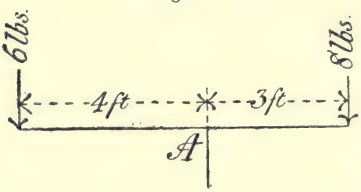
Let us take these in order.

The Principle of Moments is: that when a body is at rest, or in equilibrium, under the action of two or more forces, the tendency of all those forces which would go to cause rotation in one direction round a given point is exactly balanced by the tendency of all the forces which would go to cause rotation in the opposite direction.

This is best illustrated by a simple example. Let two weights of 8 lbs. and 6 lbs. respectively (see Fig. 1) be

supposed to rest at either end of a rod 7 ft. long, which is suspended from a point A, 3 ft. from the larger and 4 ft. from the smaller weight. It will be seen by the diagram that the weight of 8 lbs. is tending to cause rotation round A in the direction in which the hands of a clock move, while the tendency of the weight of 6 lbs. is to cause rotation in the opposite direction round the same point. These tendencies, therefore, are opposite in direction, and they are also equal in amount, for it has been found, as the result of numberless experiments which have never been known to fail (every time a butcher weighs a piece of meat he makes such an experiment), that the tendency to rotate is ascertained by multiplying the weight by the perpendicular distance from the point of rotation to the line in which the weight acts. In the case under consideration the weights act vertically downwards, so the perpendicular distances are the horizontal measurements along the rods, and the tendency of the larger weight to cause rotation is—

Fig. 1.



$$3 \text{ ft.} \times 8 \text{ lbs.} = 24 \text{ ft.-lbs.},$$

and similarly the tendency of the smaller weight to cause rotation is—

$$4 \text{ ft.} \times 6 \text{ lbs.} = 24 \text{ ft.-lbs.}$$

Thus 24 ft.-lbs. is in each case *the moment of the weight about the point of rotation.* It will be observed that feet and pounds are here taken as the units in both cases; but inches or yards, and ounces or tons might just as readily have been adopted, so long as the same units

were used throughout the particular calculation under discussion.

There is but one principle involved in the PARALLELOGRAM, TRIANGLE AND POLYGON OF FORCES. The first is shown in Fig. 2, in which two forces, A B and C B, are supposed to be acting in the directions shown by the arrows towards the point B, the line A B representing to a scale of weights (so many pounds or tons to the inch) the magnitude, as well as the direction, of the force acting from A to B, and so on. Then, by completing the

Fig. 2.

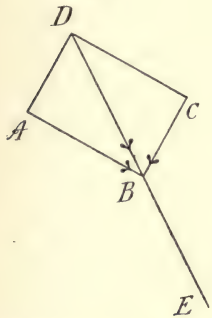
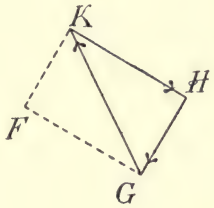


Fig. 3.



parallelogram A B C D, of which A B and C B are adjacent sides, it has been found that a single force acting in the direction of the diagonal from D to B, and equal to this diagonal in amount, as determined by scale measurement, would have the same effect upon a particle placed at B as would the two forces A B and C B acting together. This principle, again, like that of moments, has been proved by numberless experiments which have never been known to fail.

Of course, if the force D B in amount, and acting from D to B in direction, be opposed by an exactly equal force

E B, acting in exactly the opposite direction, from E to B, the particle at B will be held in equilibrium; and accordingly, if the three forces A B, C B and E B act upon the particle, the two forces A B and C B being known together to have the same effect as the one force D B, then also the particle is in equilibrium.

The triangle of forces is only used as a more simple way of expressing the same thing (see Fig. 3). In this the forces A B, C B and E B, which, as already said, hold the particle at B in equilibrium in Fig. 2, are represented by equal and parallel forces K H, H G and G K respectively, which, as will be seen by the arrows, act in sequence, as if a person were running from point to point, and form the closed triangle K H G. The remainder of the parallelogram of which they form half is dotted on the diagram, which parallelogram K H G F will be found to be similar in all respects to that in Fig. 2.

In the same way, the polygon of forces is but an extension of the triangle. Suppose the forces A O, B O, O C, D O and O E to be acting upon a particle at O (Fig. 4) in the directions designated by the arrows, then by drawing lines parallel and equal to these, in sequence, following the direction of the arrows, a closed polygon will be formed if the particle be held in equilibrium by these forces. In the case under consideration, the line F G represents the force A O, to which it is equal and parallel, G H represents B O, H K represents O C, and so on, the lines forming the complete closed polygon F G H K L F, representing in sequence each of the forces holding the particle at O in equilibrium.

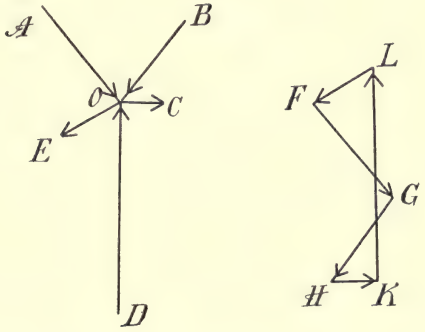
The Centre of Gravity of a body is that point round which equate all the tendencies to rotate caused by the force of gravity acting upon the various particles of which the body is made up. Assuming a body of homogeneous

substance and of uniform weight throughout (which is near enough to the truth in most cases for all practical purposes), this point has been determined for most simple-shaped bodies.

In cubes, rectangular or polygonal prisms having an even number of equal sides, spheres, cylinders, &c., the centre of gravity is at the geometric centre.

In triangles (by which is meant thin triangular prisms, as, of course, no body can have a centre of gravity which has not weight)—in triangles, then, the centre of gravity

Fig. 4.



is found by bisecting each side and joining to the opposite apex, the point of intersection being the c.g., and this being two-thirds of the length of each of these lines from its apex.

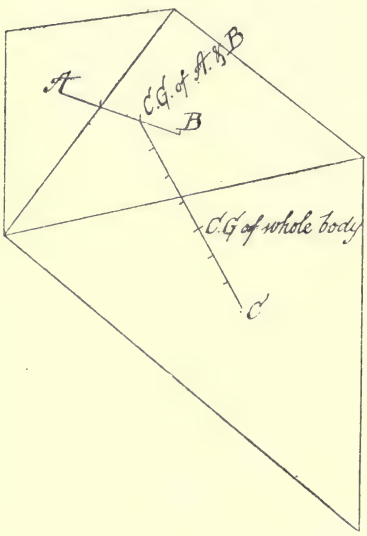
In pyramids and cones the centre of gravity is situated at a point upon the right axis, distant three-fourths of its length from its apex.

Almost all practically occurring cases can be reduced to one or other of these forms, or to combinations of them.

When finding the centre of gravity of a combination of

these forms, it is necessary to first find the c.g. of each simple form separately, and its weight (which may be supposed to be amassed at its c.g.), as is shown in the simple example given in Fig. 5. Here the weight of the triangle A is supposed to be 1 lb., which is collected at its centre of gravity; and similarly the weight of the triangle B is supposed to be 2 lbs., and that of the triangle C 4 lbs., each collected at its c.g.

Fig. 5.



First, join A and B by a straight line, and as the sum of the weights A and B = 3 lbs., therefore divide the line A B into three parts. The centre of gravity of the whole figure A + B will be where shown, at a distance of one-third of the line A B from the end B, which is the more heavily weighted.

That this is true is proved by taking moments round this point, when—

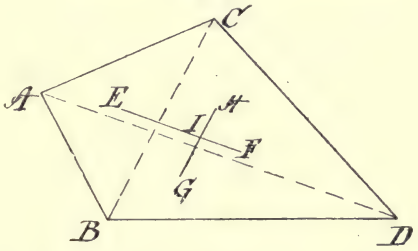
$$1 \text{ lb.} \times \tfrac{2}{3} \text{ A B} = 2 \text{ lbs.} \times \tfrac{1}{3} \text{ A B}$$
$$= \tfrac{2}{3} \text{ A B ft.-lb. in either case,}$$

showing that the tendencies to rotate round the c.g., due to gravity, equate.

Similarly, the whole weight of 3 lbs. being now concentrated at this c.g., this point can be joined to the point C, the joining line being divided into 7 parts (the sum of the weights being 7 lbs.), and the c.g. of the whole body is then found to be 3 of these parts along this line as measured from C—the point which is most heavily loaded.

The centre of gravity of an irregular quadrilateral figure can be readily found diagrammatically, as shown in Fig. 6. The c.g. of the triangle A B C is found at E, and that of the triangle B C D at F, and the points E and F are joined. Similarly, the c.g. of the triangle A B D is

Fig. 6.

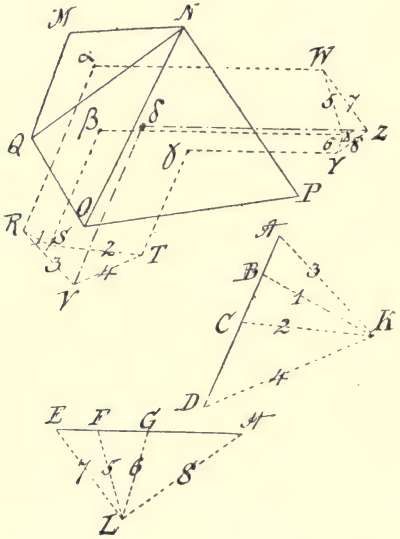


found at G, and that of the triangle A C D at H, and the points G and H are joined. The intersection of the lines E F and G H at the point I gives the c.g. of the whole figure A B D C.

There is another diagrammatical way of determining the c.g. of a prism of irregular section, which is sometimes exceedingly useful, though apparently complicated. This is exemplified in Fig. 7, in which the figure M N P O Q is an irregular polygon made up of the triangles M N Q, N Q O and N O P, the weights of which are collected at α, β and γ respectively, which are the centres of gravity of the triangles.

Through α, β and γ draw three parallel lines—α R, β S and γ T, and draw another line at side, by way of load line, also parallel, in which make A B equal to the weight located at α, to any scale; B C equal to the weight located at β, to the same scale; and C D equal to that located at γ. Then assume any pole K, and join K B,

Fig. 7.



calling this line 1. Draw another line, also known as line 1, parallel to this one, between the lines α R and β S, from R to S. Similarly join K C, calling this line 2, and draw another line 2 parallel to it from S to T, cutting γ T in T; then join K A with line 3, and draw another line 3 parallel to it from R; and finally join K D with line 4, and draw another line 4 parallel to it from T,

cutting line 3 in V. The c.g. of the whole figure will now lie somewhere in a line V δ drawn through V parallel to α R, β S and γ T.

The operation has now to be repeated, drawing α W, β X and γ Y parallel to one another in some new direction (say, horizontally) and laying down a new load line E F G H parallel to them, in which E F represents to scale the weight located at α, F G the weight located at β, and G H that located at γ. Take a new pole L, and join L F in line 5 and L G in line 6, drawing other lines 5 and 6 parallel to them, connecting W X and X Y. Again join L E and L H in lines 7 and 8, and draw lines 7 and 8 respectively parallel to them from W and Y, meeting in Z. The c.g. of the whole figure must again lie in the direction of a line Z δ, drawn through Z parallel to α W, β X and γ Y, and therefore can only lie at the point δ, where V δ and Z δ meet.

In very many calculations, in which forces actuated only by gravity, and so acting vertically downwards, are concerned, only the first half of this construction is necessary, the weights α, β and γ being taken as acting vertically, and the distances A B, B C and C D set down vertically. A case in which this occurs is shown in Fig. 13, and the *modus operandi* explained in the next chapter.

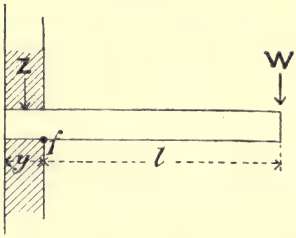
CHAPTER II.

LOADS ON SUPPORTS.

THE primary object of cantilevers, girders, bridges and roofs is to convey loads from their points or points of application to the points of support. In doing this various stresses are set up in the different members of these structures; but before the amount of these can be ascertained with any degree of accuracy, it is necessary, as will soon become apparent, to determine the proportion of the loads borne by each support.

In the case of cantilevers it is evident that, there being but one support, the whole weight must be borne by that support, whether the weight be fixed in one spot or divided into many smaller weights, or evenly distributed along the length of the cantilever.

Fig. 8.



The load itself, however, is, in the case of a cantilever, but a small proportion of what has to be carried by the support. Were not the weight W (see Fig. 8) counterbalanced by another weight, Z, the cantilever would overturn about the fulcrum, *f*. The weight Z necessary to prevent this can be readily ascertained, the cantilever now forming

a lever of the first order, by taking moments about f, remembering that Z may be considered to be applied at the centre of gravity of the surface pressed, or, in the example before us, at the centre of the wall. Thus, to produce equilibrium—

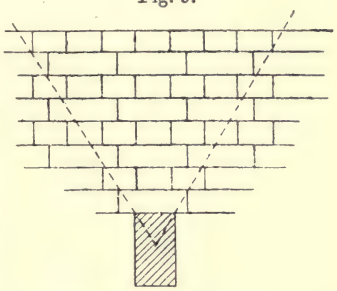
$$Z \times \frac{y}{2} = W \times l,$$

or

$$Z = \frac{2\,W\,l}{y}.$$

A considerably greater counterbalancing weight than Z, as ascertained above, should in all cases be provided for the sake of stability; but where a cantilever is built into a wall, this need not all lie directly over the cantilever, as all the bricks or stones which would be lifted in case of its overturning may be included in the load (see Fig. 9). This allowance varies with the bond employed; but as a general rule all material which would be inclosed by an equilateral triangle having the centre of the cantilever for its apex (Fig. 9) may be safely considered as forming part of Z, no additional allowance being made for the adhesion of the mortar.

Fig. 9.



It is also a common plan, where a cantilever is expected to be subjected to a comparatively heavy end load, to carry it right through the wall and well into the building upon the other side, anchoring it down at its far end. The arm y of the lever being thus increased in length

the counterbalancing weight Z may be proportionally reduced.

When the load on the cantilever is distributed (Fig. 10), it may be considered to act at its centre of gravity, and the distance of its application from f becomes $\frac{l}{2}$. Then—

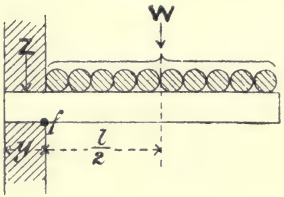
$$Z \times \frac{y}{2} = W \times \frac{l}{2},$$

or

$$Z = \frac{W\,l}{y}.$$

In either case the weight Z can be made up in many ways, as the exigencies of each case demand, and the total weight borne by the support becomes W* + Z.

Fig. 10.



When a fixed load is borne by a girder at the centre of its span, common sense tells us that half is carried by each support; but the proof of this depends again upon the principle of moments, and, as the proof is the same when W rests at any other spot than the centre, such a case is shown in Fig. 11. The girder may be considered as a lever, working round the fulcrum f to produce a certain pressure on the right-hand abutment A. By taking moments round f, to produce equilibrium we must have—

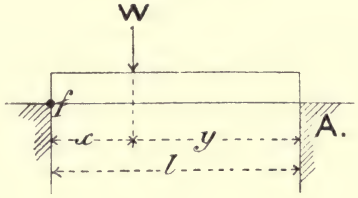
$$\text{Pressure on A} \times l = W \times x;$$

$$\text{or Pressure on A} = \frac{W\,x}{l}.$$

* In this, as in most works on this subject, W is taken to represent a fixed load, or the total of a distributed load, w the load per unit of length (generally per foot), and l the length, in the same unit.

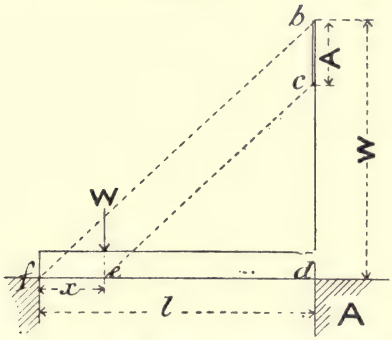
This can be shown to scale graphically, as in Fig. 12. Set up W to a scale of so many tons or pounds to the inch over the abutment A. Join the extremity *b* of this

Fig. 11.



line to *f*. From *e* (a point vertically under the point of application of W) draw *e c* parallel to *f b*; *c b* will then represent the pressure on A to the same scale as that to

Fig. 12.



which *d b* represents W. The reason for this is that *d b f* and *d c e* are similar triangles, and so the sides *b d* and *f d* are cut proportionately by the parallels *f b* and *e c*. That is—

$$b\,c : f\,e :: b\,d : f\,d,$$

or, in other words,

$$b\,c : x :: \mathrm{W} : l;$$

which is the same as saying,

$$b\,c \times l = \mathrm{W} \times x;$$

or

$$b\,c = \frac{\mathrm{W}\,x}{l};$$

proving $b\,c$ to represent the pressure on A, as obtained by the previous reasoning.

Where there are several loads on a girder, each can be considered separately as above, and the totals on each abutment added together; or else all can be treated in one single diagram by a somewhat curious application of the polygon of forces.

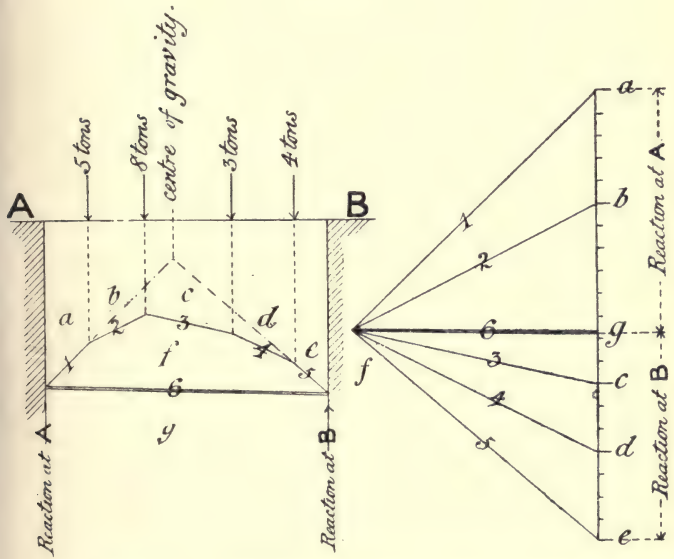
Suppose a beam or girder, resting on the supports A and B (Fig. 13), to carry loads of 5, 8, 3 and 4 tons respectively where shown, the beam in this case being delineated by a single line. Place arrows pointing downwards where each load is applied, to denote the direction in which it acts, and arrows pointing upwards under the point at which the effect of the loads is transmitted to each abutment. Letter the spaces between these arrows in alphabetical order, commencing at the left-hand side, the space between the upward arrow under A and the downward arrow over the load of 5 tons being lettered *a*; that between the loads 5 and 8 tons being lettered *b*; and so on.

As the load of 5 tons lying between the spaces *a* and *b* acts *vertically downwards*, set off at the side, to a scale of weights as before described, a distance representing 5 tons on a line drawn *vertically downwards* from *a* to *b*. Proceed in the same manner, setting off a distance representing 8 tons vertically downwards from *b* to *c*, to signify the load of 8 tons acting vertically downwards between the spaces *b* and *c*, and so on, taking care to

make the letters on sketch and diagram tally for ease of reference. By this means all the loads on the girder can be set down to scale; and it is only then required to divide the whole vertical line *a e* into two parts to denote the proportions carried by the two abutments.

Assume any point *f* outside the line *a e*, and join *a f*, *b f*,

Fig. 13



c f, *d f* and *e f*. Drop vertical lines through the centres of each load (on the sketch), and also through the points A and B, where the loads are transmitted to the abutments. Assume any point in the line drawn thus through A, and from this point draw a line parallel to the line *f a* (on diagram) until it meets the vertical line through the load of 5 tons. Place another letter *f* on the opposite side of

this new line to the space *a*, to represent the space in the centre of a polygon of which this line is one side, and it will be seen that there is a line between the spaces *f* and *a* upon the sketch parallel to the line *f a* on the diagram. From the termination of the line between the spaces *f* and *a* draw another between the spaces *f* and *b*, parallel to the line *f b* on the diagram, and terminating under the load of 8 tons, and proceed in this manner until the abutment B is reached. Join the extremities of these lines under the abutments A and B, inclosing *f* (by the line which is drawn double on sketch), and letter the space outside this and between the two upward arrows, *g*. The polygon now formed could be taken to represent in outline a truss such as would be in equilibrium under the assumed loads; and, by drawing through the point *f* on diagram a line *f g* parallel to the double line between the spaces *f* and *g* until it cuts the vertical line *a e*, the proportion of the total load borne by each abutment can be measured off, that carried by A being equal to the weight as measured by the original scale of weights represented by *a g*, and that carried by B being equal to the weight represented by *g e*.

The loads and the proportions borne by the abutments are now represented by the closed polygon *a b c d e g* on the diagram, the pressures *e g* and *g a* on the abutments being supposed to act *upwards*, as indicated by the arrows on the sketch in the first instance. This upward action or, more correctly speaking, passive resistance of the supports is an important factor in many calculations, and is generally known as *the reaction of the supports*. It is exactly equal at each support to the proportion of the load borne by that support, but is necessarily opposite to it in direction in order to cause equilibrium.

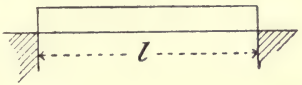
In Fig. 13, and in all other drawings throughout this work in which reciprocal diagrams are used, lines which

have to be drawn parallel to one another upon sketch and diagram are similarly numbered.

This reaction of the supports is supposed to occur at the centre of that portion of the girder which rests on the support, and in all cases of calculation affecting beams or girders it is usual to consider the span as from centre to centre of bearings, this being sufficiently near in most practical cases to be used instead of the correct centre of pressure of the reaction; and this distance is known as the "effective span." In diagrams this is usually shown as in Fig. 14, l being the "effective" span from centre to centre of bearings, and not the clear span, as would appear at first sight.

When part of a beam bears a distributed load, while the remainder is unloaded, such distributed load must be treated as a number of small fixed loads, in the manner shown in Fig. 13; but when a beam carries a uniformly distributed load throughout its length, it is evident that half of this load is carried by each support, or in other words that the reaction of each support is equal to half of the total load, and no further calculation or diagram is necessary.

Fig. 14.



While upon this subject, it may be well to note that the centre of gravity of a number of loads resting on a girder can be ascertained as shown by dotted lines in Fig. 13, by continuing the lines between the spaces f and a, and that between the spaces f and e, until they meet, which they will do vertically over or under the centre of gravity of the loads.

As proof that the point thus ascertained is the centre

of gravity of the loads, it would only be necessary to assume that their total,

$$5 + 8 + 3 + 4 = 20 \text{ tons},$$

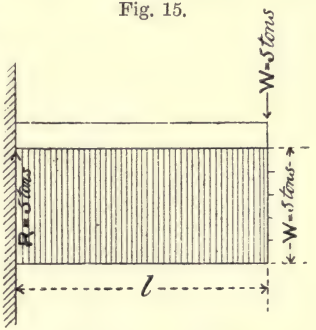
is there located, between spaces *a* and *e*. Then setting *a e* vertically downwards on the load line as 20 tons to the scale of weights, the pole *f* and the lines 1, 5 and 6 would be drawn as before, when the point *g* would be found to be in the same position as before, giving the same reactions as before.

CHAPTER III.

VERTICAL SHEARING STRESSES IN CANTILEVERS AND BEAMS.

The opposition of the two forces spoken of in the last chapter—the load borne by a beam or cantilever, and the reactions of the supports—causes in the beam a stress similar to that caused in a piece of thread when exposed to the cutting edges of a pair of scissors, which stress is known as "shearing." Scissors, however, are made sufficiently powerful to overcome the resistance which thread offers to being shorn, while beams have to be designed to contain sufficient material in their webs (if they be of the flanged type) to resist the shearing effect of the load and reaction.

Fig. 15.



In the case of a cantilever carrying an end load (Fig. 15), the reaction R and the load W are equal to one another; but their lines of action, though parallel, are opposite in direction. From this it follows that the member which enables these two forces to oppose one another, and so to produce equilibrium, is subject at each point throughout

its entire length to a shearing stress equal to either one of them—not to the sum of the two, for where two equal opposing forces of any kind meet, the stress produced is the measure of one of these forces only, because there could be none at all unless each force were opposed by an equal and opposite one, either active or passive. This can be shown graphically, as in Fig. 15, by setting down a line under W equal to W (here assumed as 5 tons) to any scale of weights, and completing the shaded parallelogram. Ordinates drawn from beneath the cantilever to the lower limit of the parallelogram will give the shearing stress, to the same scale, at any point.

Fig. 16.

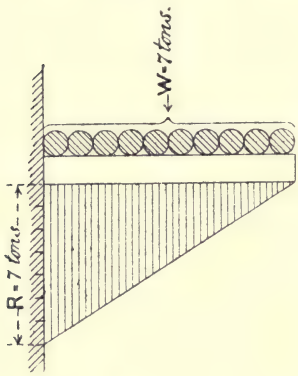
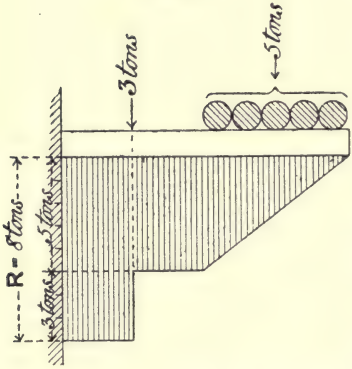


Fig. 17.



When a cantilever carries a distributed load, the shearing stress is only equal to the total load at the point of support, and uniformly diminishes as the reaction is taken up by one portion of the load after another, until at the

end of the cantilever there is no shear at all. This can be shown graphically, as in Fig. 16.

When a cantilever extends beyond the point of application of the load, there is, of course, no shearing stress beyond that point, save that caused by the weight of the cantilever itself; and this, though occasionally of considerable importance in practical work, is rarely taken into account in purely theoretical investigations.

When a cantilever has to bear two or three different loads, the shearing diagrams have to be combined, as in Fig. 17, where a cantilever is represented as bearing a fixed load of 3 tons at one quarter of its length from the support, and a distributed load of 5 tons spread over the half of its length furthest from the support.

A single fixed load on a *beam* causes a shearing stress at all points between its point of application and either support, which is equal to the reaction at that support. The reasoning for this is the same as that in the case of the cantilever; at every point between the point of reaction and the point of application of the load, the reaction is endeavouring to tear its way through the beam to reach the portion of the load which equilibrates it.

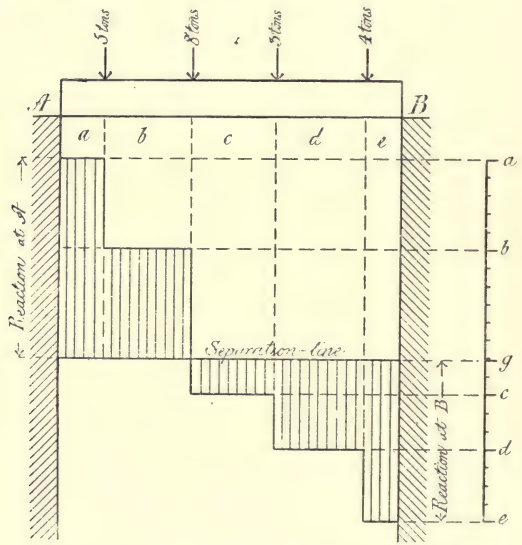
When a beam carries several loads the same principle applies—the reactions endeavour to tear upwards towards the loads. This is shown diagrammatically in Fig. 18, in which the same loading is assumed as in Fig. 13. The same load line is used, though to a different scale of weights, and the position of g upon it is similarly found, the working not being again shown, however, for the sake of clearness. Thus again $e\ g$, as measured to a scale of weights on the load line, represents the value of the reaction at B, and $g\ a$ the value of the reaction at A.

A line, which for convenience is generally horizontal,

called the "Separation Line," may now be drawn from g on the load line right across the span on the sketch.

If another line parallel to the Separation Line be now drawn from a on the load line to the line of reaction under A on the sketch, the value of the reaction under A will now be laid down to scale on the line of reaction.

Fig. 18.



This will also be the measure of the vertical shear at the abutment A, and throughout the extent of the space a, until the load of 5 tons be met which separates space a from space b. Thus ordinates from the Separation Line, drawn within the space a to the horizontal line from a on the load line, denote the shearing stresses at all points within the space a.

When the load of 5 tons is met which separates space *a* from space *b*, the shear is reduced to the extent of that 5 tons. This is shown diagrammatically by drawing a line parallel to the Separation Line from *b* on the load line across the space *b* on the sketch, when ordinates from the Separation Line to this new line, within the space *b*, denote the shears at all points within the space *b*.

These, as well as the shears within the space *a*, are shears towards the left-hand abutment A.

When the load of 8 tons between *b* and *c* is met, the shears towards A are again reduced—to the extent of 8 tons; and this again is shown diagrammatically by drawing a line parallel to the Separation Line from *c* on the load line across the space *c* on the sketch; and again ordinates from the Separation Line to this new line, within the space *c*, denote the shears at all points within the space *c*. In this instance, however, the ordinates have to be drawn downwards from the Separation Line instead of upwards as hitherto, for the load of 8 tons between *b* and *c* is greater than the amount of shear within the space *b*, and the difference occurs now as a shear towards the right-hand abutment B.

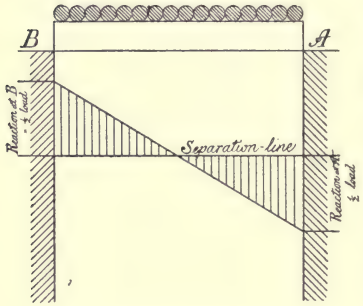
Similarly the loads of 3 tons between *c* and *d*, and of 4 tons between *d* and *e*, are picked up in turn, and lines parallel to the Separation Line drawn from *d* and *e* on the load line across the spaces *d* and *e* on the sketch; resulting in the anticipated discovery that the shears within the space *e* are equal to the reaction at B.

It will be noticed that ordinates drawn above the Separation Line denote shears towards the left-hand abutment A, and those drawn below the Separation Line denote shears towards the right-hand abutment B.

In order to ascertain the shears along a beam carrying an evenly distributed load, of which perhaps the most

perfect example is the weight of the beam itself, it is only necessary to carry this argument to its logical conclusion. As already stated (see p. 17), in such a case each reaction is equal to half of the total load. The shear at each abutment is equal to the reaction at that abutment, and uniformly diminishes towards the middle of the beam as each portion of the load is picked up; with the result shown in Fig. 19, where it will be seen that there is no shear at the centre; and a similar effect is produced when two equal loads are placed at equal distances from the centre of a beam, there being in such a case no shear between their points of application.

Fig. 19.



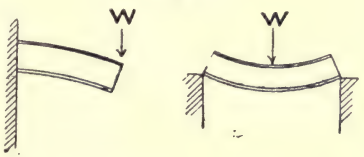
It may be mentioned, as a matter of theory if of little practical worth, that a load causes no shear immediately below its point of application, though shear occurs at even an infinitesimal distance to one side or other of that point. This, however, accounts for the fact that a rivet which is put into shear by the opposite action of two plates which it joins, will fail, if at all, between the plates and not opposite either of them, its particles being caused to slide past one another by the action of shearing stress.

CHAPTER IV.

STRESSES IN FLANGES.

In addition to the shearing stress which is calculated to be borne by the web, the flanges of flanged structures have to bear direct tension and compression. This is evident, as shown at Fig. 20, for a cantilever tends to bend under a load so as to place the lower flange in compression and the upper in tension, while a beam tends to bend in such a way as to place the lower flange in tension and the upper in compression. It will be found upon investigation that in any vertical section the tension in the one flange is equal to the compression in the other, and that there is a line along the web at approximately half its depth (the actual position varying according to the design of the girder), known as the neutral axis, where there is no stress. In practice it is usual to consider that the whole of the tension and compression is borne by the flanges, and that the web has only to be made sufficiently strong to resist the shear.

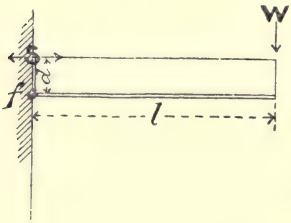
Fig. 20.



In order to ascertain the stresses in the flanges, it is first necessary to assume the length (or span) l, and the

depth d. The exigencies of the particular case under discussion will generally determine l, while d is usually assumed as some fraction of l, varying as circumstances will permit, from 1/12 to 1/20. These being given, the remainder is a matter of leverage. The double line in Fig. 21 shows the bent lever set in action by the weight W, acting at the end of a cantilever, and causing the tensional stress S in the top flange at the point nearest the support—where it is greatest. The fulcrum of this bent lever may be assumed at f, provided that the neutral axis be through the centre of the web, and round this point moments may be taken. The moment $S \times d$ must, to produce equilibrium, be equal to the moment $W \times l$, generally known as the *bending moment*. These can be equated thus—

Fig. 21.



$$S \times d = W \times l;$$

or, dividing both sides by d,

$$S = \frac{W\,l}{d}.$$

Of course, in the above it is necessary that whatever units be adopted for l and W, the same units must be adopted for d and S respectively. That is, if l be taken in inches, d must be taken in inches also; and if W be taken in tons, the stress S will also be ascertained in tons.

Correctly speaking, the long arm, l, of the lever should be taken along the neutral axis, the bending moment

being equated by the moment of inertia, or the sum of resistances of the two flanges multiplied by their mean distances respectively from the neutral axis; but the above explanation is the simpler one to follow, and produces practically the same result.

If it be required to determine the stress at any intermediate spot, it can be ascertained, as shown in Fig. 22, by assuming a bent lever round the fulcrum f'

Fig. 22.

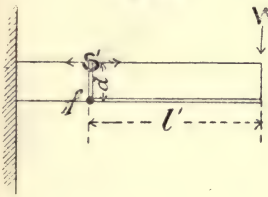
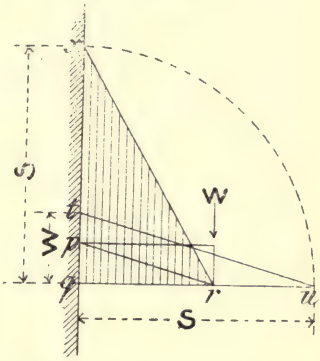


Fig. 23.



and taking the stress and bending moments round f'. Then—

$$S' \times d = W \times l';$$

or

$$S' = \frac{W \times l'}{d}.$$

As l' (Fig. 22) is less than l (Fig. 21), therefore S′ is less than S. This proves that the greatest stress is at the support. Under the weight, l is reduced to *nil*, when the stress also becomes *nil*; and the value of S will be found to vary uniformly from *nil* under the load to its

greatest value at the support. The value of S will be the same whether it represent the tension in the top or the compression in the bottom flange, at whatever spot it may be taken. It is only necessary to reverse the bent lever and to assume f in the top instead of the bottom flange to prove this.

The value of S in the formula $S = \frac{W\,l}{d}$ can be ascertained graphically, as in Fig. 23. Assuming $p\ q$ as the depth (d) and $q\ r$ as the length (l) of the cantilever, set up $t\ q$ equal to W, according to a scale of weights as previously explained. Join $p\ r$, and from t draw $t\ u$ parallel to $p\ r$, meeting $q\ r$ produced in u; then $q\ u$ is equal to S, for $p\ q\ r$ and $t\ q\ u$ being similar triangles, therefore—

$$q\ u : q\ r :: t\ q : p\ q\ ;$$

or, multiplying the extremes and means inversely and equating,

$$q\ u \times p\ q = t\ q \times q\ r,$$

that is,

$$q\ u = \frac{t\ q \times q\ r}{p\ q} = \frac{W\,l}{d},$$

which proves that $q\ u$ = S to the same scale of weights to which $t\ q$ = W.

By placing the compasses at q, and describing a quadrant with radius $q\ u$, $q\ x$ is made equal to S. Join $x\ r$, and then ordinates drawn from the bottom flange to the line $x\ r$ will denote the stresses in the flanges at any points to the same scale as that to which $t\ q$ represents W.

When a cantilever is subject to a distributed load, such load may be assumed to act at its centre of gravity (see Fig. 24). The bending moment is then but $W \times \frac{l}{2}$,

the bent lever being as shown in Fig. 24, and the equation of equilibrium obtained by taking moments round the fulcrum f becomes—

$$S \times d = W \times \frac{l}{2};$$

or

$$S = \frac{W\,l}{2\,d}.$$

If it be required to ascertain the stress at an intermediate spot, it can be ascertained as shown in Fig. 25, that portion of the load only being considered which rests between the spot at which it is required to determine the

Fig. 24.

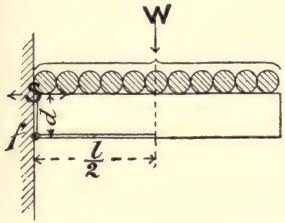
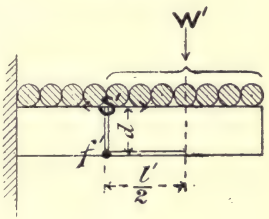


Fig. 25.



stress and the end of the cantilever. The equation of moments would then be—

$$S' \times d = W' \times \frac{l'}{2};$$

or

$$S' = \frac{W'\,l'}{2\,d}.$$

In this case, as in that of an end load, there is found to be no stress at the end of the cantilever, W and l having both diminished to *nil;* while the greatest value of S is obtained at the support p; but the intermediate variations are as the square of the distance from the

extremity of the cantilever, instead of being uniformly as that distance.

The value of S in the formula $S = \frac{W\,l}{2\,d}$ can be

Fig. 26.

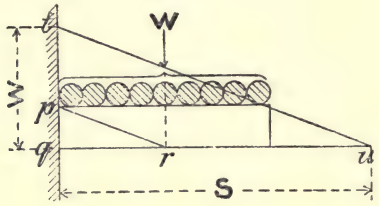
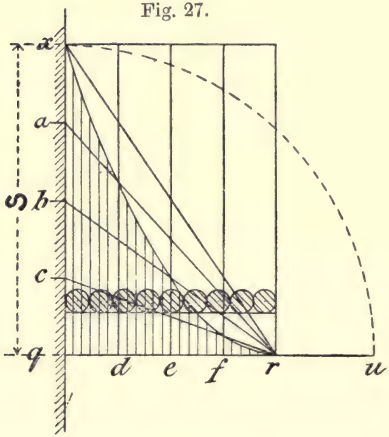


Fig. 27.



ascertained graphically, as shown in Fig. 26, in which the same letters have been adopted as in the similar Fig. 23, *r* now being placed under the centre of gravity of the total distributed load W. The same reasoning being followed

as that used with Fig. 23, it is seen that in the similar triangles $p\,q\,r$, $t\,q\,u$,

$$q\,u : q\,r :: t\,q : p\,q\,;$$

or

$$q\,u \times p\,q = t\,q \times q\,r;$$

therefore,

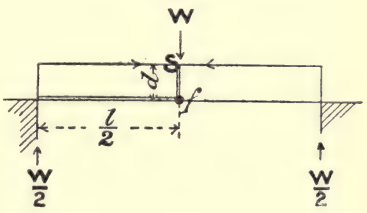
$$q\,u = \frac{t\,q \times q\,r}{p\,q} = \frac{\mathrm{W} \times \frac{l}{2}}{d} = \frac{\mathrm{W}\,l}{2\,d}\,;$$

and therefore

$$q\,u = \mathrm{S}.$$

To determine the stresses at intermediate points, set up $q\,x$ equal to $q\,u$ (or S), as shown in Fig. 27. Between x and r set up a semi-parabola, having its apex at r. This may be accomplished by dividing $x\,q$ into any number of equal parts, as at a, b and c, and $q\,r$ into the same number of parts as at d, e and f. Join a, b and c to r, and through d, e and f erect vertical lines. Join x to r by a line passing through the points of intersection, thus tracing the outline of a semi-parabola. Ordinates drawn to this from the bottom flange will give the stresses at all intermediate points to the original scale of weights.

Fig. 28.



In flanged beams, the stresses are caused by the reactions of the supports, and not by the load itself, the greatest stress being under the load. When a beam has to carry a fixed central load, the bent lever causing the maximum stress in the top flange is as shown

in Fig. 28, f being the fulcrum, and the equation of moments—

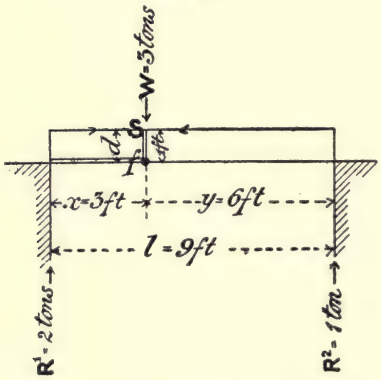
$$S \times d = \frac{W}{2} \times \frac{l}{2};$$

or

$$S = \frac{W\,l}{4\,d}.$$

If the load be not centrally placed, the stress beneath it is found in precisely the same manner, the reaction at either support being taken to produce the bending moment: whichever be taken, the stress discovered to exist in the flange at the point under the load will be the same. An example is given in Fig. 29, where the bending moment may be taken to be either $R^1 \times x$ or $R^2 \times y$; in either case, 6 foot-tons. In other words—

Fig. 29.



$$S \times d = R^1 \times x, \text{ or } R^2 \times y;$$
$$\therefore S = \frac{R^1 \times x}{d}, \text{ or } \frac{R^2 \times y}{d}$$
$$= \frac{2 \times 3}{1}, \text{ or } \frac{1 \times 6}{1}$$
$$= 6, \qquad \text{or } 6.$$

The stresses at intermediate points are found as shown in Fig. 30, R^1, the reaction at the abutment A, causing

the stresses at all points between that abutment and the point of application of W, and R^2 causing the stresses at all points between the abutment B and the point of application of W. In Fig. 30 it is assumed that the flange stresses are required to be ascertained at a point at which

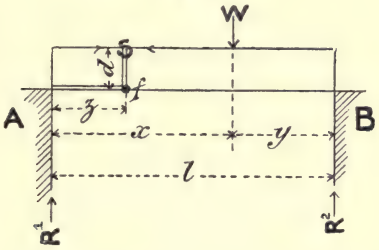
$$S \times d = R^1 \times z;$$

or

$$S = \frac{R^1 \times z}{d}.$$

The value of S is evidently greatest when z is greatest

Fig. 30.



—that is, when S is required under the load—and least when z is least, viz. at the abutment, when z is *nil*, and S consequently is *nil* also. The stress varies uniformly according to the value of z

The value of the stresses under a fixed load, whether it be central or otherwise, can be ascertained graphically, as shown in Fig. 31, in which the same letters have been used as in Figs. 23 and 26. From the bottom flange under the load $t\ q$ is set up equal to R^1 to any scale of weights, p and r are joined by a straight line, and through t a line $t\ u$ is drawn parallel to $p\ r$. S is now denoted by $q\ u$ to the same scale as that to which $q\ t$ was set up.

This depends upon similar reasoning to that adopted before, for $p\ q\ r$ and $t\ q\ u$ being similar triangles,

$$q\ u : q\ r :: t\ q : p\ q ;$$

or

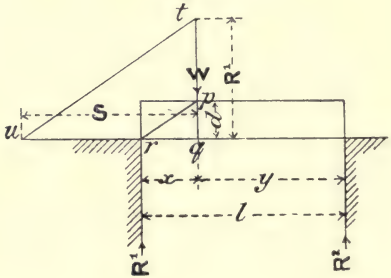
$$q\ u \times p\ q = t\ q \times q\ r$$

$$\therefore q\ u = \frac{t\ q \times q\ r}{p\ q}$$

$$= \frac{\mathrm{R}^1 \times x}{d}$$

and therefore $q\ u = \mathrm{S}$, for this agrees with the result arrived at above when considering Fig. 29.

Fig. 31.



To determine the stresses at intermediate points, set up $q\ m$ vertically from the bottom flange under the load equal to $q\ u$ or S. (See Fig. 32.)

Join $m\ r$ and $m\ n$, and then ordinates set up from the bottom flange to $m\ r$ and $m\ n$ will give the stresses at any intermediate points.

It is easiest to ascertain the stresses in a flanged beam due to a distributed load by making an assumption. Two or three different assumptions will lead to the

desired result; but perhaps the easiest to comprehend is that shown in Fig. 33. The proportion of the load on the half-beam nearest to the abutment A,

Fig. 32.

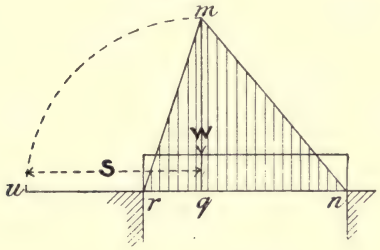
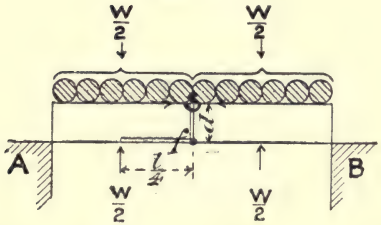


Fig. 33.



instead of reacting at A is assumed to react at its centre of gravity.

By this means the lever causing stress at the centre becomes as shown, and—

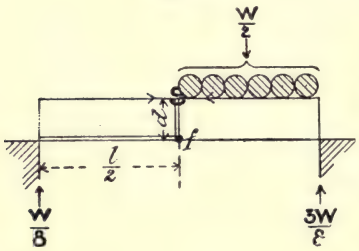
$$S \times d = \frac{W}{2} \times \frac{l}{4},$$

or

$$S = \frac{W\,l}{8\,d}.$$

The same result can be arrived at by supposing each half of the load $\left(\frac{W}{2}\right)$ to act at its centre of gravity as a fixed load. Then the reaction at the further abutment due to this half-load is $\frac{W}{8}$ (see Fig. 34), causing stress at the centre with a leverage of $\frac{l}{2}$, this stress being half the stress which would be produced were the girder loaded throughout its full length, or $\frac{S}{2}$, as a similar stress would be produced by the load lying on the other half of the girder. Thus,

Fig. 34.



$$\frac{S}{2} \times d = \frac{W}{8} \times \frac{l}{2},$$

or

$$\frac{S}{2} = \frac{W\,l}{16\,d},$$

or

$$S = \frac{W\,l}{8\,d},$$

the same result as before ascertained.

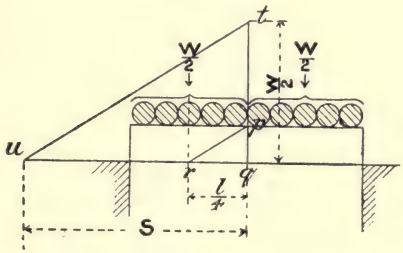
The stresses at intermediate points can be similarly found by assuming the portion of the load on either side of the point at which the stress is required to be found to be a fixed load acting at its centre of gravity, ascertaining the stresses produced by each of these loads at the required point, and summing them.

The stress is greatest at the centre, and is *nil* at the supports, but varies between these points in a com-

pound ratio, which is sufficiently indicated in the last paragraph.

These stresses can be ascertained graphically, as shown in Fig. 35, in which the same letters have been used as in the previous examples (Figs. 23, 26, and 31), $t\ q$ being set up over the centre equal to $\frac{W}{2}$ to scale of weights, and $q\ r$ being taken equal to $\frac{l}{4}$. Then $q\ u$ is equal to the stress in the centre. The reasoning is exactly similar to that

Fig. 35.



adopted before, for $p\ q\ r$ and $t\ q\ u$ being similar triangles,

$$q\ u : q\ r :: t\ q : p\ q,$$

or

$$q\ u \times p\ q = t\ q \times q\ r;$$

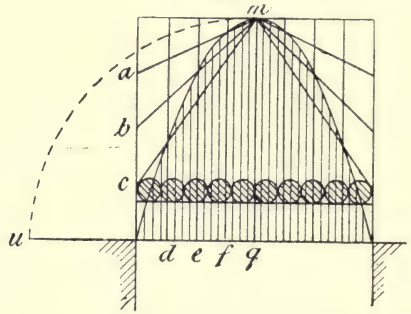
$$\therefore\ q\ u = \frac{t\ q \times q\ r}{p\ q} = \frac{\frac{W}{2} \times \frac{l}{4}}{d} = \frac{W\ l}{8\ d} = S.$$

The intermediate stresses are more readily ascertained geometrically than arithmetically (see Fig. 36). Set up $q\ m$ vertically over the centre of the bottom flange equal to $q\ u$ or S as previously ascertained, and describe the parabola as shown (in the same manner as explained in Fig. 27). Ordinates drawn from the bottom flange to the

outline of this parabola will give the stresses at any intermediate points.

When a load is distributed over only a part of a girder, the remainder being unloaded, it is usual to assume the load to be a fixed load acting at its centre of gravity. This can be done without great error when the load covers only a small portion of the length of the girder, while such error as occurs is on the side of safety; but when the load and the proportion of the girder which it covers are considerable, it is wisest to ascertain the stresses at all

Fig. 36.

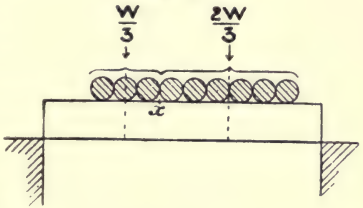


points correctly, according to the method explained later on with Fig. 43, or else as shown in Fig. 37, by taking several points as x, assuming the load lying on each side of x to act at its centre of gravity, ascertaining the stresses at x due to each of these parts of the load, and summing them. By doing this at several points, a stress outline can be set up which will delineate the stresses at all points, as is done in Figs. 32 and 36. It will be noticed, however, that the curved outline will only extend over that portion of the beam which is loaded; over the unloaded portion the outline will be exactly the same as it would

have been had the load been a fixed load acting at its centre of gravity. The error of assuming the load to be fixed is thus seen to be slight, and only to affect the portion of the beam which is loaded, while its rectification is only of importance as saving the employment of unnecessary material in the flanges directly under the load.

When, as often happens, a beam has to carry several loads at different points, the stresses due to each have to be ascertained separately, and then combined at all points before a girder can be designed to resist them. To do this arithmetically, by ascertaining the stresses at several given points from each load in succession, and adding these stresses together to ascertain the total stress at each of these points, is tedious in the extreme. Graphically considered, the problem is an easy one.

Fig. 37.

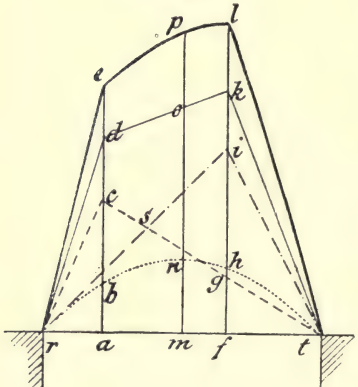


In Fig. 38 an outline of short dashes ($r\,c\,t$) represents the stress outline due to one fixed load, the dot-and-stroke outline $r\,i\,t$ represents the stress outline due to another fixed load, and the dotted outline $r\,b\,n\,h\,t$ represents the stress outline due to a distributed load, such as the weight of the girder itself. Of the triangles $r\,c\,t$ and $r\,i\,t$, the portion $r\,s\,t$ is common to both—which means that the stresses which it represents occur twice over, and that the figure $r\,c\,s\,i\,t$ plus the figure $r\,s\,t$ represents the stresses due to the two fixed loads. The triangle $r\,s\,t$ has therefore to be combined with the figure $r\,c\,s\,i\,t$ to represent these

stresses in one figure. This can be done by making $i\,k$ equal to $f\,g$, and $c\,d$ equal to that portion of the line $a\,c$ which lies within the triangle $r\,s\,t$, and by joining $r\,d\,k\,t$. The figure $r\,d\,k\,t\,i\,s\,c\,r$ can now be readily proved, by the theory of triangles on equal bases and with equal altitudes, to be equal to the triangle $r\,s\,t$. In other words, the figure $r\,d\,k\,t$ represents the total stresses due to the two loads.

The stress outline $r\,b\,n\,h\,t$, due to the distributed load, can similarly be added to this combined diagram by making $d\,e$ equal to $a\,b$, $o\,p$ equal to $m\,n$, and $k\,l$ equal to $f\,h$, and by joining $r\,e\,p\,l\,t$ in the firm curved outline shown, which outline represents the total stresses at all points due to the two fixed loads and the distributed load.

Fig. 38.

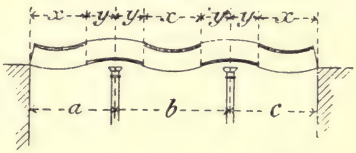


In all cases yet investigated it has been assumed that the girders merely rest upon the supports without being fixed in any way. When the ends of girders are fixed, they are placed under an entirely new set of conditions; but it is so difficult to absolutely fix them over single spans that they are almost invariably assumed to be supported only even when held in position by bolts or by a superincumbent weight. When, however, girders are so built as to form one continuous structure over several spans,

the portion over each central span becomes absolutely fixed at its ends.

In Fig. 39 an example is given of a girder which is continuous over three spans *a*, *b* and *c*. Of these spans the portions *x x x* are girders with the top flange in compression and the bottom in tension: while the portions *y y y y* are cantilevers with the top flange in tension and the bottom in compression. The stresses on *x* are found as usual for girders, with a span equal to *x*. One of the abutments of *x* is the end of *y*, which carries its proportion of the load on *x* as an end load, besides any load to which it may itself be subject, the stresses being ascertained in the usual manner for cantilevers. The greatest stress will, under almost all conditions, be found to be over the intermediate supports, while there will be no stresses at the points where the ends of *x* rest on the ends of *y* (which points are called "points of contraflexure").

Fig. 39.



It has been by the careful adoption of this principle of continuous girders that such immense bridges as that over the Forth at Inchgarvie have been rendered possible.

The stresses in the flanges of supported (i.e., not fixed) girders can be ascertained at all points in an exceedingly simple manner by means of a reciprocal polar diagram, similar in principle to that shown in Fig. 13. Taking the simplest case—that of a cantilever with an end load—and assuming this load W to be equal to 8 tons, acting vertically downwards, set this 8 tons to a scale of weights vertically downwards from a point *a*. (See Fig. 40.) From *a* set off *a o* horizontally, equal to *d*, the depth of

the cantilever, and join o b. From r, a point in the bottom flange of the cantilever immediately under the load, draw r x parallel to o b on the diagram. Ordinates

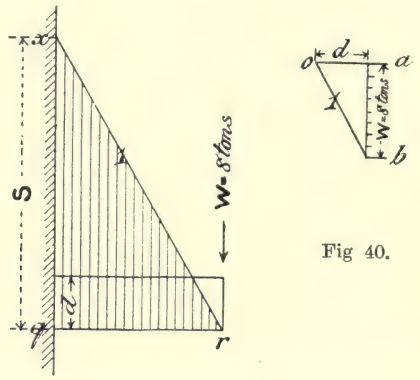


Fig 40.

from q r to x r will give the value of the stresses in the flanges at any point.

The proof of this is simple, for the triangles x q r, b a o are similar to one another, and therefore—

$$x\,q : q\,r :: b\,a : a\,o,$$

or

$$x\,q \times a\,o = b\,a \times q\,r,$$

or

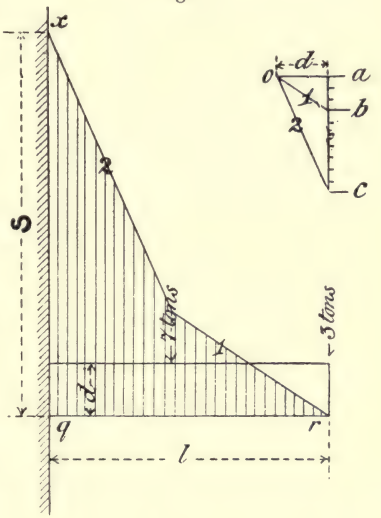
$$x\,q = \frac{b\,a \times q\,r}{a\,o} = \frac{\mathrm{W}\,l}{d},$$

proving that $x\ q = \mathrm{S}$ according to the recognised formula. Moreover, the triangle x q r thus obtained is identical with the triangle x q r as ascertained in Fig. 23.

If the distance d on the diagram be set out to equal one foot, to the scale to which the sketch is drawn, the

distance $x\ q$ will represent the Bending Moment in foot-tons; for if $d = 1$ and $x\ q = \frac{W\ l}{d}$, then $x\ q = W\ l$. This holds for all the remaining diagrams in this chapter, and forms an exceedingly convenient method of ascertaining the B M when, as is frequently the case, that and not S is required to be known. It is merely a matter of graphic arithmetic, for the flange stress is always equal to $\frac{B\ M}{d}$, and the B M is always equal to S d.

Fig. 41.



In a simple case like the foregoing the advantages of this method are not very conspicuous; but more complicated cases are much simplified by its adoption. Suppose, for example, that the cantilever carries two fixed loads of 3 and 7 tons respectively at different points (see Fig. 41), acting vertically downwards. According to a scale of weights, set vertically down from a point a the distance $a\ b$, equal to 3 tons, and from b set down $b\ c$, equal to 7 tons. From a set off $a\ o$ horizontally, equal to d, the depth of the cantilever. Join $o\ b$ and $o\ c$.

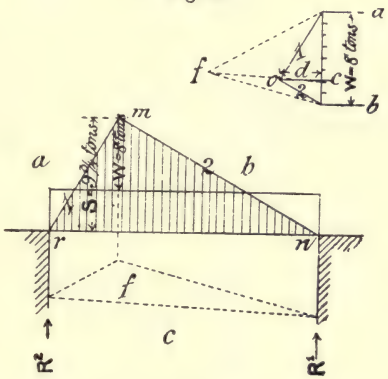
From the point r vertically under the load of three tons, and in the bottom flange of the cantilever, draw a

line parallel to *o b* until a vertical line through the centre of gravity of the load of seven tons is reached; and from that point draw a line parallel to *o c*, meeting the abutment at *x*. Ordinates drawn from *q r* to the outline from *x* to *r* will give, to the scale of weights employed in the diagram, the value of the stresses in the flanges at all points; and a little consideration will show that the figure *x q r* merely contains the sum of the triangles which would have been obtained had each of the weights been considered separately. Cases in which three or four, or any greater number of loads have to be considered, can be treated in a precisely similar manner, until, by considering a distributed load to be an aggregate of an infinite number of infinitesimal loads, an outline similar to that shown in Fig. 27 can be arrived at. In order to obtain an accurate result, however, in this, as in all other systems in which diagrams are employed, the most absolutely perfect draughtsmanship is essential. In important calculations it is therefore desirable to employ a large scale and a fine line, so as to reduce the risk of errors to a minimum.

A simple application of this system to a supported girder can now be taken, in which a weight of eight tons acts vertically downwards at some fixed point. (See Fig. 42). From any point *a* set *a b* vertically downwards, equal to eight tons to any scale of weights. From *b* set *b c* vertically upwards, equal, to the same scale, to R^1, the reaction of the right-hand abutment. Then *c a* will be equal to R^2, the reaction of the other abutment: these reactions being ascertained by the dotted polar and funicular diagrams, as explained in connection with Fig. 13. From *c* set *c o* horizontally equal to *d*, the depth of the girder. Join *o a* and *o b*. From *r*, the point in which the bottom flange of the girder meets the left-hand abutment,

draw $r\ m$ parallel to $o\ a$, and from n draw $n\ m$ parallel to $o\ b$. If accurately drawn, $r\ m$ and $n\ m$ will meet in m, vertically over the centre of gravity of the weight W; and ordinates drawn from $r\ n$ to the outline $r\ m\ n$ will give the value of the stresses in either flange at any point, to the same scale of weights as that employed in setting down the diagram. Very little consideration will show that the triangle $r\ m\ n$ thus obtained is identical

Fig. 42.

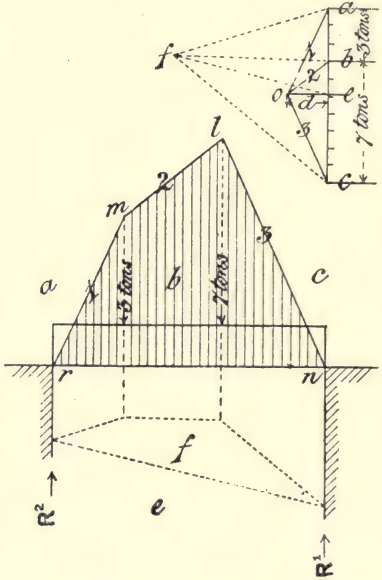


with the triangle $r\ m\ n$ obtained in Fig. 32. The proof is obvious and need not be repeated.

The advantages of this method of computing stresses are, in the girder as in the cantilever, more considerable in complicated than in simple cases. Suppose that the girder carry two loads of 3 and 7 tons respectively at different points. (See Fig. 43.) From a point a set vertically downwards $a\ b$ to any scale of weights equal to 3 tons, from b set down $b\ c$ equal to 7 tons, and from c set upwards $c\ e$ equal to R^1, the reaction at the right-hand abutment; $e\ a$ will then be equal to the reaction at the

other abutment. These reactions, as before, are ascertained by the dotted polar and funicular diagrams. From e set off horizontally $e\ o$ equal to d the depth of the girder, and join $o\ a$, $o\ b$ and $o\ c$. From r draw $r\ m$ parallel to $o\ a$ until a vertical line through the centre of gravity of

Fig. 43.



the load of 3 tons is reached at m. From m draw $m\ l$ parallel to o till a vertical line through the centre of gravity of the load of seven tons is reached at . From l draw $l\ n$ parallel to $o\ c$. If the drawing be accurate, the point n should correspond with the corner of the right-hand abutment. Ordinates from $r\ n$ to the outline $r\ m\ l\ n$ will give the values of the stresses in either flange at any

point to the scale of weights employed. It will be found on trial that the polygon $r\ m\ l\ n$ corresponds with that obtained by summing the triangles arrived at by considering each of the loads separately, as explained in connection with Fig. 38. The system is applicable to any number of loads, until, by considering a distributed load as an aggregate of many small loads, an outline similar to that shown in Fig. 36 would be arrived at.

In all these diagrams, Figs. 40, 41, 42 and 43, it is permissible, and it is often a distinct convenience to set out d on the diagram equal to twice or thrice the depth of the girder to the scale of the sketch—with the result that S will equal one-half or one-third respectively of the flange stress.

Similarly, if d on the diagram be set out equal to 2 ft. or 3 ft., to the scale of the sketch, S will equal one-half or one-third, as the case may be, of the Bending Moment in ft.-tons.

CHAPTER V.

RECTANGULAR BEAMS.

In calculations relating to rectangular beams, such as those of timber, it is usual to employ the inch as the unit of dimension, and either the lb. or the cwt. as the unit of weight. Of these last, the cwt. is generally the more convenient and will be employed here, for although the use of the smaller unit would enable calculations to be made with greater exactitude, this is not necessary when dealing with a material which varies so greatly as does timber, even of the same kind, in strength and in freedom from defects.

It is also usual to work calculations with the B.W., or "breaking weight," and not with the safe load.

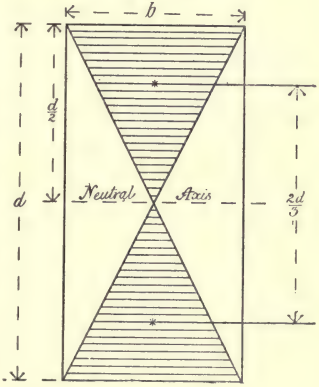
As has already been seen, in all cases Bending Moment (B.M.) must equate with the Stress Moment, in flanged structures, and the method of finding the B.M. under any conditions of loading have been explained (see Chapter IV.), it being only necessary to substitute inches and cwts. as the units instead of feet and tons as there used. The only new point, consequently, is the determination of the Stress Moment, and incidentally to discover the "flange" of a beam of rectangular section. Such a beam is shown in section in Fig. 44.

When bent, as under the influence of a superimposed load, the outer fibres at top and bottom are put into either tension or compression, as explained with reference to

Fig. 20, while somewhere between these is a layer of fibres known as the neutral axis, which is unaffected. For convenience this may be assumed to lie midway between top and bottom of the beam, though it does not necessarily do so, this depending upon the comparative strength of the material to resist compression and tension.

If now diagonal lines be drawn across the section, as is done in Fig. 44, the horizontal shading within these will diagrammatically represent the grading of the strains in the various fibres, from a maximum at top and bottom to zero at the neutral axis. In other words, all except the shaded portions might be cut away, and yet enough material be left to do the whole work; and advantage of this is frequently taken when joists are tusk-tenoned into a beam. These shaded portions form, theoretically, the flanges of the beam, under full stress throughout.

Fig. 44.



The sectional area of each of these triangular flanges is $\frac{b\,d}{4}$, and its ultimate resistance is ascertained by multiplying this by the ultimate resistance of the material per square inch as adjusted to compensate for the assumption of a central neutral axis and consequently of equality of resistance to tension and compression. This is known as the "modulus of rupture," and is commonly written as $f\,o$.

The ultimate resistance of each flange, or the stress upon it when subject to bending under the breaking weight (B W), is $f o \frac{b\,d}{4}$.

The theoretic depth of the beam is not its actual extreme depth, but the distance between the centres of gravity of the two triangles or flanges. This theoretic depth is consequently $\frac{2\,d}{3}$.

Therefore the Moment of Ultimate Resistance, corresponding with the Stress Moment (S d) used throughout Chapter IV., is $f o \frac{b\,d}{4} \times \frac{2\,d}{3}$,

$$= f o \frac{b\,d^2}{6},$$

$$= \frac{f o}{6} b\,d^2.$$

This can be simplified by substituting K for $\frac{f o}{6}$, when it becomes K $b\,d^2$; and in all cases this equates with the maximum bending moment produced by the breaking weight, or, *always*,

$$\text{B M} = \text{K}\,b\,d^2.$$

Tabulating, the simple cases then work out as follows :—

Cantilever, end load (see p. 26)—

Bending Moment.

$$\text{B W} \times l = \text{K}\,b\,d^2, \quad \therefore \text{B W} = \text{K}\frac{b\,d^2}{l}.$$

Cantilever, distributed load (see p. 29)—

$$\text{B W} \times \frac{l}{2} = \text{K}\,b\,d^2, \quad \therefore \text{B W} = 2\,\text{K}\frac{b\,d^2}{l}.$$

Beam, central load (see p. 32)—

Bending Moment.

$$\frac{B\,W}{2} \times \frac{l}{2} = K\,b\,d^2 \quad \therefore B\,W = 4\,K\frac{b\,d^2}{l}.$$

Beam, distributed load (see p. 35)—

$$\frac{B\,W}{2} \times \frac{l}{4} = K\,b\,d^2 \quad \therefore B\,W = 8\,K\,\frac{b\,d^2}{l}.$$

In these equations, B W and K are in cwts., and all dimensions are in inches.

The values of K (which, it may be remembered, is $\frac{f\,o}{6}$, or one-sixth of the modulus of rupture) are as follows for ordinary specimens of the various materials, specially selected specimens showing higher values:—

K for Elm	= 9 cwts.	...	C = 3 cwts.
K ,, Baltic fir	= 10 ,,	...	C = $3\frac{1}{3}$,,
K ,, British oak	= 13 ,,	...	C = $4\frac{1}{3}$,,
K ,, Teak	= 19 ,,	...	C = $6\frac{1}{3}$,,
K ,, Greenheart	= 26 ,,	...	C = $8\frac{2}{3}$,,
K ,, Cast iron	= 46 ,,	...	C = $15\frac{1}{3}$,,
K ,, Wrought iron	= 68 ,,	...	C = $22\frac{2}{3}$,,
K ,, Mild British Steel	= 100 ,,	...	C = $33\frac{1}{3}$,,

The original experiments upon rectangular beams having been made upon pieces of wood of 1 in. square section suspended upon knife edges 1 ft. apart, the actual breaking loads thus obtained were taken as constants, known as C, and a series of formulæ evolved for the simple cases, very similar to the reasoned equations given above—giving, in fact, the same results when C is taken as equal to $\frac{K}{3}$. Under these circumstances the formulæ work out as follows, for the simple cases only:—

For cantilever, end load ... $B\,W = C\frac{b\,d^2}{4\,L}$.

For cantilever, distributed load... $B\,W = C\,\frac{b\,d^2}{2\,L}$.

For beam, central load ... ... $B\,W = C\,\frac{b\,d^2}{L}$.

For beam, distributed load ... $B\,W = 2\,C\,\frac{b\,d^2}{L}$.

In these formulæ, B W and C are in cwts., b and d are in inches, and L is in feet. They do not provide for any other than the simple cases, there being no ready equation with the B M.

In practical work it is more frequently required to ascertain what beam is needed to carry a given load, than, given the beam, to find what load it will sustain; so, by way of an example, it may be well to take such a case, assuming a span of 18 ft., an evenly distributed load of 4 cwts. per ft. run (or a total distributed load of 72 cwts.), in addition to a load of 24 cwts. located at 6 ft. from one of the supports. These, multiplied by 4 as a factor of safety, give loads of 288 cwts. distributed, and 96 cwts. located to be borne at the time of fracture. At the two points at which the highest B M is likely to be found, viz., at the centre of the beam and under the located load, the values of the B M in cwt.-inches due to these breaking loads are as follows :—

At Centre.

Due to distributed load... $\frac{B\,W}{2} \times \frac{l}{4} = \overset{\text{Cwts.}}{144} \times \overset{\text{Ins.}}{54} =$ 8,316

Due to located load ... Reaction of further support from load— $\times \frac{b}{2} = \overset{\text{Cwts.}}{96} \times \frac{1}{3} \times \overset{\text{Ins.}}{108} =$ 3,456

Total = 11,772

Under Located Load.

Due to distributed load—

$$\left\{\begin{array}{l}(12\text{ ft. @ }16 \times \frac{1}{3} \times \text{ Cwts. } 72 \text{ Ins.}) \\ +(6\text{ ft. @ }16 \times \frac{1}{6} \times 144)\end{array}\right\} = \left\{\begin{array}{l}(64 \times \text{ Cwts. } 72 \text{ Ins.}) \\ +(16 \times 144)\end{array}\right\} = 6{,}912$$

Due to located load—

$$96 \times \tfrac{1}{3} \times \text{Cwts. } 144 \text{ Ins.} \quad \ldots \quad \ldots \quad \ldots \quad \ldots = 4{,}608$$

$$\text{Total} = 11{,}520$$

Of these, the B M at centre is the greater, and has to be met by K $b\,d^2$.

If oak is used, K = 13 ;
and therefore, if K $b\,d^2 = 11{,}772$,

$$b\,d^2 = \frac{11{,}772}{13},$$

$$= 906 \text{ (nearly).}$$

For a beam to be of the best proportions, its breadth should $= \frac{5\,d}{7}$ and therefore $b\,d^2 = \frac{5d^3}{7}$.

Therefore $$\frac{5\,d^3}{7} = 906,$$

$$\therefore\ 5\,d^3 = 6{,}342,$$

$$\therefore\ d^3 = 1{,}268,$$

$$\therefore\ d = \sqrt{1{,}268},$$

$$= 11 \text{ inches (nearly),}$$

and as $b = \frac{5\,d}{7}$ $\therefore\ b = \frac{55}{7}$ (nearly),

$$= 8 \text{ inches (nearly).}$$

Consequently an oak beam 11 inches deep and 8 inches wide may be safely used under the given conditions.

Theoretically, the correct forms for beams of rectangular cross section throughout should be those given, in each case in alternative, in Figs. 45 to 52. Practically it is generally found to be more economical to make rectangular in all directions, and of their maximum section uniformly throughout.

Correct theoretical forms for rectangular beams to assume.

Cantilever with end load.

Fig. 45.

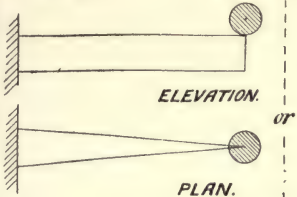
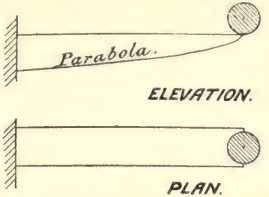


Fig. 46.



Cantilever with distributed load

Fig. 47.

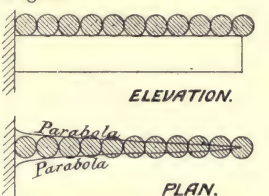
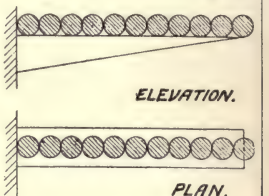


Fig. 48.



Beam with single load.

Fig. 49.

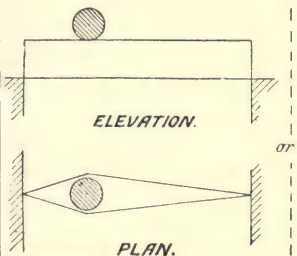
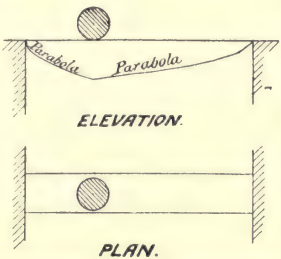


Fig. 50.



Beam with distributed load.

Fig. 51.

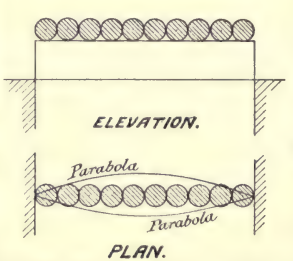
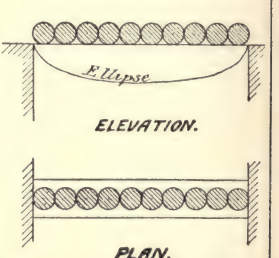


Fig. 52.



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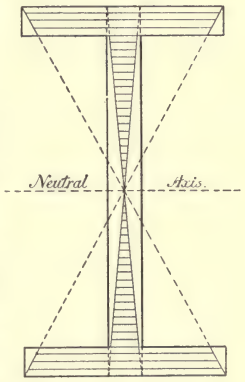
CHAPTER VI.

METHOD OF DESIGNING A ROLLED STEEL JOIST.

For all ordinary spans, up to even so much as 35 or 40 ft., occurring in structural work, it is usual to employ steel joists. These have stout flanges and thin webs, the section approximating to that shown in Fig. 53, and given accurately in Fig. 56.

It will be seen that Fig. 53 is but a modification of Fig. 44, the flanges and web being treated each as part of a rectangular beam, and the shaded portions representing the useful area to resist stress. The theoretical depth would be the distance between the centre of gravity of the shaded portions above and below the neutral axis.

Fig. 53.

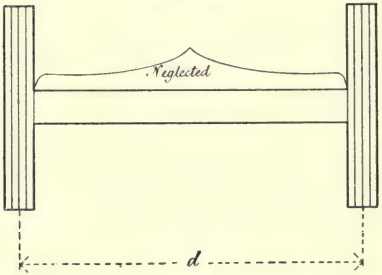


Accurate calculating on this basis is, however, rarely undertaken. The results obtained by considering the flanges as rectangles of their average thickness which are under uniform stress, while neglecting the web, with the result that the depth is taken as that between the centres of the flanges, as shown in Fig. 54, are almost identical,

and calculations on this basis are much more simple. Any variation from exactitude, negligible in any case, would be by no means so great as is the variation of section which even the same rollers will give from time to time, all manufacturers demanding a small margin within which to work.

By way of illustration, the smaller girder C D, shown

Fig. 54.



in plan in Fig. 55 as resting on a wall at D, and carried at C by the larger girder A B, may be assumed to carry an evenly distributed load of $\frac{1}{2}$ ton per foot run over a span of 25 ft., this including the weight of the joist itself, in addition to a load of 5 tons at a point 10 ft. from D.

The B M's due to these loads are as follows :—*

At Centre.

ft.-tons.

Due to distributed load... $\frac{W\,l}{8} = \frac{12\frac{1}{2} \times 25}{8} = \frac{312\frac{1}{2}}{8} = 39\frac{1}{16}$

Due to fixed load ... R at C $\times\ 12\frac{1}{2} = 2 \times 12\frac{1}{2} = 25$

Total $= 64\frac{1}{16}$

* The B M's could be ascertained graphically almost as accurately. Students would find it a good exercise to check the calculations in that way.

Under Fixed Load.

Due to distributed load—

ft.-tons.

$$\left.\begin{array}{l}\text{(R at D due to } 7\tfrac{1}{2}\text{ T} \times 10)\\ +\text{(R at C } \text{,, ,, } 5\text{ T} \times 15)\end{array}\right\} = \left\{\begin{array}{l}(\tfrac{3}{10} \times 7\tfrac{1}{2} \times 10)\\ +(\tfrac{1}{5} \times 5 \times 15\ )\end{array}\right\}$$

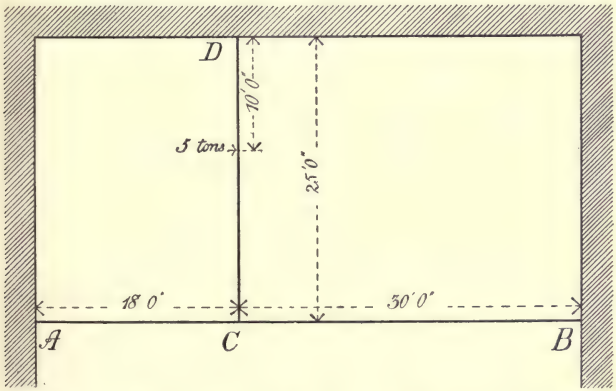
$$= 22\tfrac{1}{2} + 15 = 37\tfrac{1}{2}$$

Due to fixed load—

R at C $\times$ 15 = 2 $\times$ 15...	=	30
Total	=	$67\tfrac{1}{2}$

Thus the B M is greatest under the fixed load, where it is $67\tfrac{1}{2}$ ft.-tons; or $67\tfrac{1}{2} \times 12 = 810$ inch-tons.

Fig. 55.



This has to be met by the moment of resistance at the safe stress per square inch of the material—that is, by the area of the effective section (the shaded portion of the diagrams, Figs. 53 or 54), in inches multiplied by the effective depth between the centres of gravity of these sections multiplied by the safe stress. With good

British steel, the safe stress is $7\frac{1}{2}$ tons per square inch; so that

$$\text{B M} = \text{M R (Moment of Resistance) in sq. ins.} \times 7\tfrac{1}{2},$$

$$\text{and } \therefore \frac{\text{B M}}{7\frac{1}{2}} = \text{M R}.$$

Substituting the ascertained value (810 inch-tons) for the B M, this equation becomes

$$\text{M R} = \frac{810}{7\frac{1}{2}},$$

$$= 108 \text{ square inches.}$$

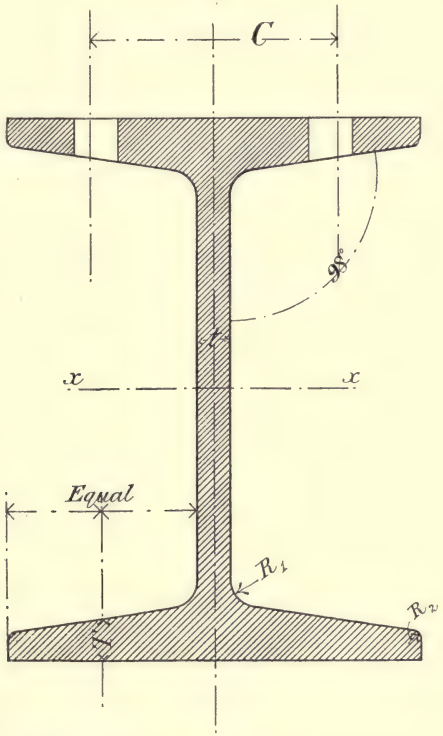
In all modern tables of steel joists the moments of resistance in square inches are given, and a joist has to be selected whose M R is at least as great as is required. Thus the smallest Standard Joist having a moment of resistance of at least 108 square inches is No. 28 B.S.B., 18 ins. by 7 ins., and weighing 75 lb. per foot run. Its M R is 127·6 inches, which allows a large margin of safety. (See Table of Standard Steel Joists given on p. 61.)

It is imperative that the depth should be at least one-twentieth of the span, else deflection may occur. In this case $\frac{1}{20}$ span is 15 inches, so that the depth of the selected joist (18 ins.) is more than sufficient.

Were it necessary, for the sake of obtaining head-room beneath it or for any other reason, to reduce the depth of the joist to its minimum, a joist 15 ins. deep would have had to be selected, two being placed side by side, or else steel plates being riveted to the flanges, the moments of resistance of which would compensate for that which is lacking in the joist itself. It is generally more economical under such circumstances to use two or more joists than

to go to the expense of drilling rivet holes and riveting. In the case now under consideration, for instance, two Standard Joists, No. 25 B. S. B., each 15 ins. deep and

Fig. 56.



having a M R of 57·06 sq. ins. would suffice. They would, however, cost more than the deeper single joist, No. 28 B. S. × B., previously selected, each weighing 42 lb. per foot run, and the pair 84 lb. per foot run, whereas

the single larger joist only weighs 75 lb. per foot run; but this would to a certain extent be compensated by their being easier to handle.

When joists have to carry distributed loads only, no calculations beyond those necessary to determine the total loads are necessary, as all manufacturers issue tables showing what evenly distributed loads their various joists are calculated to carry over given spaces (see p. 62). These data, as well as the Moments of Resistance in square inches, are given in the following list of British Standard Sections culled from the section book published by Messrs. Dorman, Long & Co.

The Standard Section is shown in Fig. 56, the sizes of the various parts being given, with corresponding reference letters, in the table on next page :—

LIST OF BRITISH STANDARD BEAMS.

PROPERTIES OF SECTIONS.

Reference No.	Size. Inches.	Weight per foot. Lbs.	Area. Sq. inches.	Diagram. web. t.	Diagram. flange. T	Diagram. radius. R1	Diagram. radius. R2	Moments of Resistance about x—x inch. Sq. inches.	Cross Centres of Holes C inches.
B.S.B. 30 ...	24 × 7½	100	29·4	·6	1·07	·7	·35	221·1	4·5
,, 29 ...	20 × 7½	89	26·17	·6	1·01	·7	·35	167·0	4·5
,, 28 ...	18 × 7	75	22·06	·55	·928	·65	·325	127·6	4
,, 27 ...	16 × 6	62	18·23	·55	·847	·65	·325	90·71	3·5
,, 26 ...	15 × 6	59	17·35	·5	·88	·6	·3	83·85	3·5
,, 25 ...	15 × 5	42	12·35	·42	·647	·52	·26	57·06	2·75
,, 24 ...	14 × 6	57	16·76	·5	·873	·6	·3	76·12	3·5
,, 23 ...	14 × 6	46	13·53	·4	·698	·5	·25	62·92	3·5
,, 22 ...	12 × 6	54	15·88	·5	·883	·6	·3	62·58	3·5
,, 21 ...	12 × 6	44	12·94	·4	·717	·5	·25	52·55	3·5
,, 20 ...	12 × 5	32	9·41	·35	·55	·45	·225	36·66	2·75
,, 19 ...	10 × 8	70	20·6	·6	·97	·7	·35	68·98	4·75
,, 18 ...	10 × 6	42	12·35	·4	·736	·5	·25	42·3	3·5
,, 17 ...	10 × 5	30	8·82	·36	·552	·46	·23	29·12	2·75
,, 16 ...	9 × 7	58	17·06	·55	·924	·65	·325	51·0	4
,, 15 ...	9 × 4	21	6·176	·3	·46	·4	·2	18·02	2·25
,, 14 ...	8 × 6	35	10·29	·44	·597	·54	·27	27·62	3·5
,, 13 ...	8 × 5	28	8·24	·35	·575	·45	·225	22·33	2·75
,, 12 ...	8 × 4	18	5·294	·28	·402	·38	·19	13·92	2·25
,, 11 ...	7 × 4	16	4·706	·25	·387	·35	·175	11·2	2·25
,, 10 ...	6 × 5	25	7·35	·41	·52	·51	·255	14·53	2·75
,, 9 ...	6 × 4½	20	5·88	·37	·431	·47	·235	11·54	2·5
,, 8 ...	6 × 3	12	3·53	·26	·348	·36	·18	6·736	1·5
,, 7 ...	5 × 4½	18	5·29	·29	·448	·39	·195	9·076	2·5
,, 6 ...	5 × 3	11	3·235	·22	·376	·32	·16	5·444	1·5
,, 5 ...	4¾ × 1¾	6·5	1·912	·18	·325	·28	·14	2·833	
,, 4 ...	4 × 3	9·5	2·794	·22	·336	·32	·16	3·76	1·5
,, 3 ...	4 × 1¾	5	1·47	·17	·24	·27	·135	1·834	
,, 2 ...	3 × 3	8·5	2·5	·2	·332	·3	·15	2·524	1·5
,, 1 ...	3 × 1½	4	1·176	·16	·248	·26	·13	1·106	

1. The customary margin required in rolling is 2½ per cent. over or under the specified weights, and no guarantee is given to roll sections without this allowance, while all beams are cut to a margin of 1 inch over or under specified lengths. An extra is always charged for cutting beams to within an eighth of an inch of exact length and for machining square.

2. Beams over 36 feet long are charged extra for every foot over that length.

TABLE OF SAFE DISTRIBUTED LOADS IN TONS ON BEAMS OF VARYING SPANS. LENGTH OF SPAN IN FEET.

2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	Deflection Co-efficient.	Reference No.
					92	79	69	61	55	50	46	42	39	36	34	32	30	29	27	26	25	24	23	22	·00078	B. S. B. 30
			94	83	69	59	52	46	41	38	34	32	29	27	26	24	23	22	20						·000937	,, 29
			78	64	53	45	40	35	32	29	26	24	22	21	20	18	17	16							·00104	,, 28
		73	56	45	38	32	28	25	22	20	19	17	16	15	14	13	12	11							·00117	,, 27
		62	52	42	35	30	26	23	21	19	17	16	15	14	13	12	11								·00125	,, 26
		47	35	28	24	20	18	16	14	13	12	11	10	9·5	9	8·4									·00125	,, 25
		59	47	38	31	27	24	21	19	17	16	14	13	12											·00133	,, 24
		43	39	31	26	22	19	17	15	14	13	12	11	10											·00133	,, 23
	53	52	39	31	26	22	19	17	15	14	13	12	11	10											·00156	,, 22
		40	33	26	22	19	16	14	13	12	11	10	9												·00156	,, 21
	32	30	23	18	15	13	11	10	9	8·3	7·6	7	6·5												·00156	,, 20
		53	43	34	28	24	21	19	17	15	14	13													·001875	,, 19
		35	26	21	17	15	13	11	10	9·6	8·8	8													·001875	,, 18
	30	24	18	14	12	10	9	8	7·2	6·6	6	5·6													·001875	,, 17
	44	42	32	25	21	18	16	14	12	11	10	9·8													·00208	,, 16
	22	15	11	9	7·5	6·4	5·6	5	4·5	4	3·7														·00208	,, 15
	31	23	17	14	11	9·8	8·6	7·7	7	6·3															·00234	,, 14
	25	18	14	11	9	8	7	6·2	5·5	5															·00234	,, 13
19	17	11	8·7	7	5·8	5	4·3	3·8	3·5	3·2															·00234	,, 12
15	14	9·4	7	5·6	4·7	4	3·5	3·1	2·8	2·5															·00268	,, 11
22	18	12	9	7·3	6	5·2	4·5	4																	·003125	,, 10
20	14	9·6	7·2	5·8	4·8	4·1	3·6	3·2																	·003125	,, 9
14	8·4	5·6	4·2	3·4	2·8	2·4	2·1	1·9																	·003125	,, 8
13	11·3	7·6	5·6	4·5	3·8	3·2	2·8																		·00375	,, 7
2·8	6·8	4·5	3·4	2·7	2·3	1·9	1·7																		·00375	,, 6
7	3·5	2·4	1·8	1·4	1·2	1																			·00395	,, 5
7·8	4·7	3·1	2·3	1·9	1·6	1·3																			·00469	,, 4
4·6	2·3	1·5	1·1	·91	·76	·65																			·00469	,, 3
5·3	3·2	2	1·6	1·2	1	·9																			·00625	,, 2
2·8	1·4	·92	·6	·55	·46	·39																			·00625	,,

1. The loads given in the Table are based on an extreme fibre stress of 7½ tons per square inch, being one-fourth of the average breaking stress, and are calculated on the assumption that the beams receive the usual side support as in building work. For other cases, such as concentrated, or live loads, special concentration is necessary.

2. Care should be taken in selecting beams that the load is not so great as to cause excessive deflection, which may occur if the span exceeds 20 times the depth. This limit is indicated by the zig-zag line in the table.

3. The deflection (in inches) for tabular load is found by multiplying the square of the span in feet by the co-efficient which is given for each section. If the actual load is less than the tabular load the deflection will be less in exactly the same proportion.

CHAPTER VII.

METHOD OF DESIGNING A STEEL PLATE GIRDER.

In designing a built-up plate girder, such as the larger (A B) of the two shown in Fig. 55, a few more calculations are necessary, and it is generally more convenient to ascertain the flange stresses at once than to equate bending and resistance moments. In this case, the load borne at C is known to be two-fifths of the fixed load of 5 tons, plus one-half of the distributed load of $12\frac{1}{2}$ tons, or a total of $8\frac{1}{4}$ tons; while an evenly distributed load of $1\frac{1}{2}$ tons may be assumed in addition—not by any means an excessive load for a girder over a shop front, for instance, carrying the front wall of a moderately high building.

The span, it may be noted, is assumed not to be taken in the clear, but to be the effective span between centres of bearings, which, with 2 ft. bearings (each $\frac{1}{24}$ span), gives a clear span of 46 ft., and a total length of girder of 50 ft. over all. The depth between the centres of gravity of the flanges may be assumed at 2 ft. 6 ins., or just in excess of $\frac{1}{20}$ span.

Under these circumstances, the flange stress at centre due to the distributed load of $1\frac{1}{2}$ tons per foot run is therefore, by the usual formula of $S = \frac{W\,l}{8\,d}$, equal to

$$\frac{72 \text{ tons} \times 48 \text{ ft.}}{8 \times 2\frac{1}{2} \text{ ft.}} = 172{\cdot}8 \text{ tons.}$$

Set this up over the centre of a line representing the girder in elevation (Fig. 57), and complete the parabolic outline of intermediate stresses.

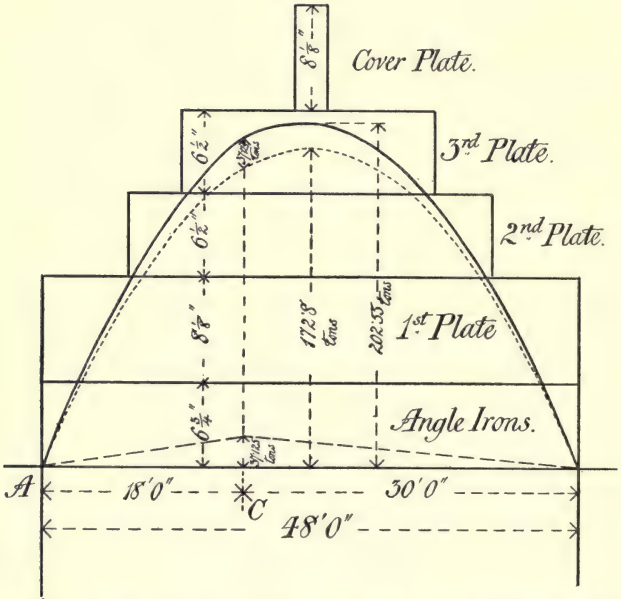
The stress in either flange over the point C is found, as explained in Chapter IV., thus :—

$$S = \frac{B\,M}{d} = \frac{\text{Reaction at A due to C} \times \text{distance A C}}{\text{depth}}$$

$$= \frac{(\frac{5}{8} \times 8\frac{1}{4} \text{ tons}) \times 18 \text{ ft.}}{2\frac{1}{2} \text{ ft.}} = \frac{92\frac{1}{16}\frac{3}{6} \text{ ft.-tons.}}{2\frac{1}{2} \text{ ft.}}$$

$$= 37{\cdot}125 \text{ tons.}$$

Fig. 57.



Set this up over C (Fig. 57) to the same scale as that used to set up the 172·8. The triangular and parabolic

outlines must next be summed, by adding together the ordinates to each from all points of the line A B, thus producing a new outline of total stresses due to both the fixed and distributed loads, as shown by the firm line in Fig. 57.

The greatest stress in either flange is thus seen to be produced over the centre of the girder, where it is equal to 172·8 tons (due to distributed load) + 29·75 tons (due to fixed load), making a total stress of 202·55 tons.

In the tension flange, therefore, at $7\frac{1}{2}$ tons per square inch, a sectional area of 202·55 ÷ 7·5, = 27 sq. ins., of steel is required to meet the stress over the centre of the girder. Besides this, rivet holes have to be allowed for, and $\frac{3}{4}$-in. rivets would almost certainly be used.

This could be made up as follows:—

	Gross Contents. Sq. ins.	Nett Contents. Sq. ins.
Two 4-in. × 4-in. × $\frac{1}{2}$-in. L-irons	$7\frac{1}{2}$	
After deducting two $\frac{3}{4}$-in. × $\frac{1}{2}$-in. rivet holes		$6\frac{3}{4}$
One 16-in. × $\frac{7}{8}$-in. plate	14	
After deducting four $\frac{3}{4}$-in. × $\frac{7}{8}$-in. rivet holes		$11\frac{3}{8}$
One 16-in. × $\frac{3}{4}$-in. plate	12	
After deducting four $\frac{3}{4}$-in. × $\frac{3}{4}$-in. rivet holes		$9\frac{3}{4}$
Nett total ...	=	$27\frac{7}{8}$

Alternatively, one $\frac{5}{8}$-in. and two $\frac{1}{2}$-in. plates might be used, with the advantage of securing more nearly uniformity in the thickness of the plates and angle irons, together with slightly greater economy of metal in adjusting their lengths—and the alternative would consequently

be adopted. After deducting rivet holes the section thus stands, as shown in Fig. 58 :—

	Nett Area.
Two 4-in. × 4-in. × $\frac{1}{2}$-in. L-irons =	$6\frac{3}{4}$ sq. ins.
One 16-in. × $\frac{5}{8}$-in. plate ... =	$8\frac{1}{8}$ „
One 16-in. × $\frac{1}{2}$-in. plate ... =	$6\frac{1}{2}$ „
One „ „ ... =	$6\frac{1}{2}$ „
Total =	$27\frac{7}{8}$ „

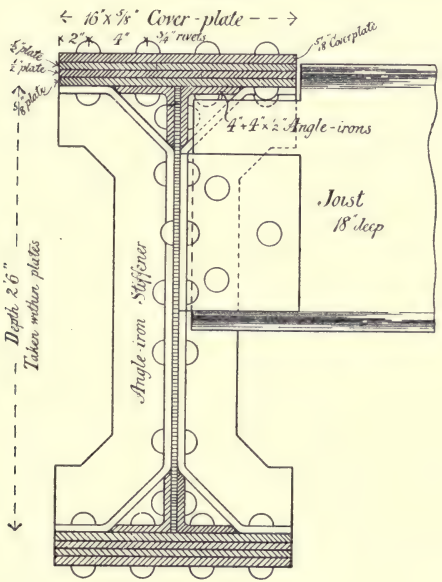
As a matter of practice, the thickest plate is usually placed next the L-irons and the compression and tension flanges are made alike in all respects.

The distance representing 202·55 tons to scale in Fig. 57, may also be taken to represent, to a different scale, the required sectional area of 27 sq. ins., and to this new scale the various sectional areas of the several plates may be laid down and the plates themselves superimposed upon the stress diagram. All they need to do is just to include the diagram, and thus their lengths and longitudinal positions may be determined. As shown in Fig. 57, the angle irons and first plate would extend throughout the entire span, partly to give a workmanlike finish and partly to provide bearing surface on the abutment below and for any wall resting upon the girder above. The second and third plates, however, would be stopped, for the sake of economy, one rivet hole beyond the plates where they overlap the firm outline of total stresses.

Angle irons are ordinarily rolled without extra charge up to 50 ft. in length, but the limit for plates is 40 ft. While the second and third plates are within this limit, it is exceeded by the first plate. Either an extra must be paid for unusual length—for plates and angle irons alike can be rolled if necessary up to a maximum length of

60 ft.—or it must be made in two lengths and a cover plate used to cover the joint. If the latter course be adopted, every possible way in which fracture may occur must be separately considered and provided against. This is a simple case, as only one plate is cut; but in larger

Fig. 58.



girders where several joints occur in the plates, the problems may become complex, and no clearer rule than this can be laid down. In ordinary constructional work, however, greater spans than that now under consideration are rarely met with, so that these difficulties do not present themselves.

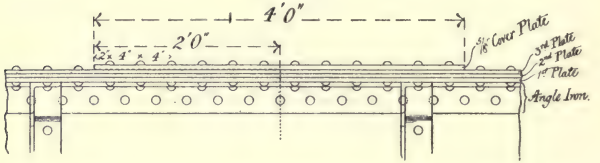
It is obviously first of all necessary that the cover

should be of the same thickness as the severed plate, that it may do the latter's work where it is cut. The cover in this instance must consequently be $\frac{5}{8}$ in. thick.

Next it is necessary that enough rivets should connect the cover and the rest of the flange to prevent fracture along the dotted line in Fig. 59. In other words, the resistance of the rivets, *on each side of the joint*, to shear must be at least equal to the resistance of the effective area (after deducting rivet holes) of the severed plate to tension.

The severed plate has, as has been already seen, a nett sectional area of $8\frac{1}{8}$ sq. ins., which, at the safe load of

Fig. 59.



$7\frac{1}{2}$ tons per square inch, would enable it to resist safely a stress of $60\frac{1}{16}$ tons. The plate being cut, this stress has to be met, on each side of the joint, by the resistance of rivets to shear. In steel, the safe allowance against shearing is 6 tons per square inch, so that a total sectional area of $10\frac{1}{6}$ ins. is necessary in the rivets. It is usual to employ $\frac{3}{4}$-in. diam. rivets, the sectional area of each of which is ·44 sq. ins.

Twenty-four such rivets are consequently necessary on each side of the joint, for

$$10\tfrac{1}{6} \div \cdot 44 = 24.$$

Reference to the section (Fig. 58) will show that four rows of rivets are constructionally necessary in each

flange, the inner rows to connect the angle irons and the flange plates, and the outer ones to bind the flange plates together. These, as is frequently though not invariably the case, would be spaced at 4-in. centres, allowing 2 ins. between the centre of the outer rivets and the edges of the flange; and longitudinally also 4-in. spacing would probably be adopted.

The cover plate, having to be long enough to include at least 24 rivets on each side of the joint, would have to be 24 ins. long on each side of it, or 4 ft. long in all; and, being cut square, would include 24 rivets (six cross rows of four each) on each side.

It is often possible to make use of surplus metal lying outside the total outline of stresses (Fig. 57) to reduce either the area of cover plate, or if there are more than one joint, the distance between joints, while in many cases a top plate can be slightly lengthened to serve the purpose of a cover by placing the joints near to its end. In fact, it is difficult to lay down any other rule for designing cover plates than that already given, that every possible way in which fracture can occur should be considered.

The greatest shear on the girder, which the web is always designed to bear, occurs at the abutment A, where it is equal to

$$\tfrac{5}{8} \times 8\tfrac{1}{4} \text{ tons} + \frac{72 \text{ tons}}{2} = 41\tfrac{5}{32} \text{ tons}.$$

At 6 tons per square inch (the safe resistance of steel to shear) this calls for a sectional area of 7 sq. ins. in the web, which is much more than met by a web of 30 in. $\times \tfrac{3}{8}$ in.—the smallest obtainable, or at least of any value against corrosion.

It will be noticed (Fig. 58) that the depth d is taken between the inner sides of the flange plates. Where,

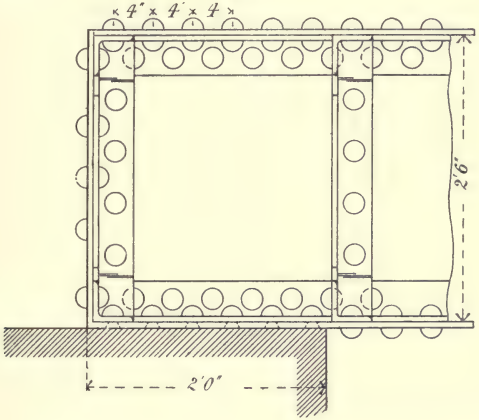
near the abutments, there is only a single flange plate, this is approximately correct as between centre and centre of flanges, of which the angle irons are considered to form part, but not the portion of web enclosed between them—a rough and ready way of compensating for the rapid diminution of the shaded area of the web, as shown in Fig. 53, as the neutral axis is approached, which is in general use. Nearer the middle of the girder this value of d is not theoretically sufficient, but any error is upon the side of safety, and it would unnecessarily complicate the calculations to assume a varying value for d at different points, where the variations would be so small as in an ordinary plate girder. If the girder were designed, however, with curved upper or lower flanges, as is occasionally though rarely the case, this would have to be done.

In Fig. 58 there is shown, besides the section of the large girder, the method of connecting the smaller joist to it, presuming that the single deeper joist is used, and that the top flanges are required to be on the same level. The cross girder, it will be noted, is connected to the web only of the main girder, its upper flange being cut away and its web notched to escape the upper flange of the larger girder. This is done to bring the load centrally on the main girder and prevent torsional stress upon its web.

The connecting rivets should contain enough metal to resist, as a shear, the proportion of weight transferred from one girder to the other. This is only $8\frac{1}{4}$ tons, calling for a sectional area in the rivets of $1\frac{3}{5}$ ins., at 6 tons per square inch. Thus three rivets of $\frac{3}{4}$-in. diam. only are needed – in single shear. Small angle connecting pieces are used, and more rivets are shown, which, moreover, are in double shear where they connect to the cross girder, they being practically rather than theoretically needed.

Stiffeners to the main girder would be introduced, as shown in Fig. 58, opposite to and under the cross girder, and also throughout the length of the main girder, not more than 5 ft. (generally about 4 ft.) apart, in order to prevent the web from buckling. They would consist of L-irons, probably 4 ins. × 4 ins. × $\frac{1}{2}$ in., bent to avoid the angle irons of the girder, and attached to the flanges by the outer rivets, and to the web ; while, if it were necessary

Fig. 60.



at any point to joint the web, T-irons would be substituted for L-iron stiffeners, the joints being made beneath them. These stiffeners cannot be calculated, as buckling, if it occur at all, would be due to some such accidental cause as laterally unequal loading.

Fig. 60 shows an elevation over the bearing, corresponding with the section given in Fig. 58. The exact dimensions and positions of the stiffeners are determined by the rivet-pitch, which forms the unit of measurement;

but a stiffener is always placed over the edge of the abutment, and another, having a backing plate attached to it of the same width as the flanges, at the girder's extreme end, to close it up. The bearing is taken as $\frac{1}{24}$ span, which is about the minimum for satisfactory work; and the rivets are countersunk on the bearing.

CHAPTER VIII.

COLUMNS AND STRUTS—WOODEN PILLARS.

Columns and struts vary in their bearing capacity, not only with their sectional area, but also greatly with their length. This variation does not depend upon mathematical principles, and it has only been possible to arrive at a formula which will give an approximately accurate result after much patient investigation of the results of a long series of experiments. Several such formulæ have been put forward from time to time, but all depend upon certain definite hypotheses. The one given here is the well-known "Gordon's Formula"; it is as easy of application and as correct as any. It is for columns of less than thirty diameters in length, and is based upon the following assumptions—viz. (1) the ends must be planed or turned so that their surfaces will be perpendicular to the axis of the pillar; (2) the bases and caps must be planed or turned to fit the ends; (3) the pillars must be set with the axes vertical and directly under the centre of the load.

Where these rules cannot be complied with, as in the struts of a roof truss, a slight modification of the formula is necessary. Under ordinary circumstances, however, where the rules apply, it has been found that

$$\text{Breaking weight in tons} = \frac{F K}{1 + C \frac{l^2}{k^2}},$$

when K = breaking weight of the materials in tons per inch, as ascertained in the usual way by crushing solid cylinders whose length is equal to twice their diameter

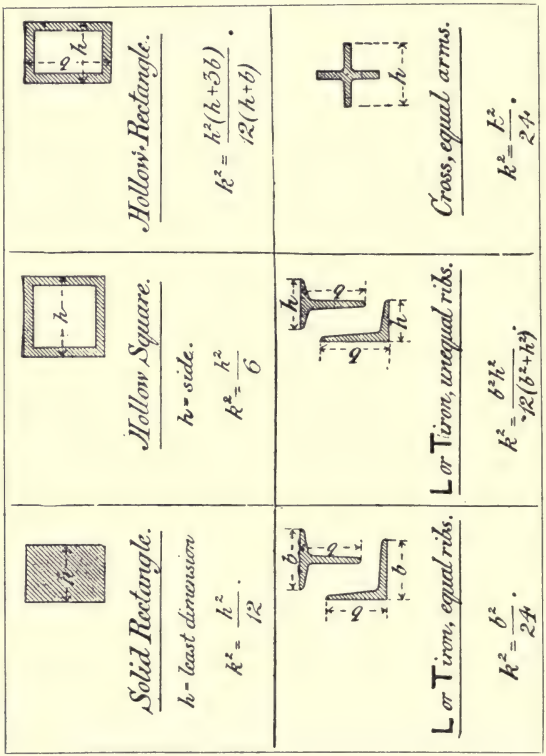


Fig. 61. Fig. 62. Fig. 63.

Fig. 64. Fig. 65. Fig. 66.

(in other words, K = 36 tons for cast iron and 16 tons for wrought iron), F = sectional area of metal, l = length in inches, C = 1/6,400 for cast iron and 1/36,000 for

wrought iron, while the value of k^2 in inches is, for various sections, that given in Figs. 61 to 70. Where one end is jointed or bears unevenly, use 2 C instead of C, and where both ends are jointed use 3 C.

If it be required to ascertain the weight which the

Fig. 67.

Channel Iron.

A = *area of flanges*
B = *area of web.*
h' is to middle of web only.

$$k^2 = k^2\left\{\frac{A}{12(A+B)} + \frac{AB}{4(A+B)^2}\right\}$$

Fig. 68.

I beams used as pillars

A = *area of flanges.*
B = *area of web.*

$$k^2 = \frac{b^2}{12} \times \frac{A}{A+B}$$

Solid Circle.

$$k^2 = \frac{d^2}{16}.$$

Hollow Circle.

$$k^2 = \frac{d^2}{8}.$$

Fig. 69. Fig. 70.

column will bear safely, the value of K must be one-sixth of the breaking weight in quiescent structures, and one-eighth of the breaking weight in structures subject to a moving load.

It is more often requisite to discover the necessary sectional area for a column which has to carry a given

load than to find out the bearing capability of a given column. In this case W, the load which it will be necessary for the columns to carry safely, is known; K must be taken as one-sixth or one-eighth, as explained above, of the breaking weight of the material per square inch, and the formula must read—

$$F = \frac{W\left(1 + C\frac{l^2}{k^2}\right)}{K}$$

EXAMPLE.

By way of an example, it may be assumed that it is required to find what hollow C I column is necessary to carry safely a load of 50 tons, its extreme height being 12 ft. Such a column, with proper base and cap plates, may be taken as being fixed at both ends, and consequently $C = \frac{1}{6400}$, while the value of k^2 is $\frac{d^2}{8}$, where d (the smallest external diameter) is not less than $\frac{1}{20}$ of the height, or, say, 8 ins. Therefore $k^2 = \frac{8^2}{8} = 8$.

Multiplying the actual load to be carried by 6, as a factor of safety, gives a breaking weight of 300 tons; and the equation stands thus:—

$$F = \frac{W\left(1 + C\frac{l^2}{k^2}\right)}{K}$$

$$= \frac{300\left(1 + \frac{1}{6400} \times \frac{144^2}{8}\right)}{36}$$

$$= \frac{300\left(1 + \frac{81}{200}\right)}{36}$$

$$= 11{\cdot}7 \text{ inches.}$$

If $\frac{3}{4}$-inch metal be used, the mean diameter of the column becomes $8 - \frac{3}{4} = 7\frac{1}{4}$ inches, and the mean circumference 22·77 inches, which, $\frac{3}{4}$-inch thick, only provides 17·08 square inches of metal, which is *more than* sufficient.

If $\frac{1}{2}$-inch metal be used, however, the mean diameter of the column becomes $8 - \frac{1}{2} = 7\frac{1}{2}$ inches, and the mean circumference 23·57 inches, which, $\frac{7}{8}$ inch thick, provides 11·79 square inches of metal, *which is enough.*

Steel Stanchions.

Steel joists are now frequently used as stanchions, erected vertically. Where such are employed it is better to consult a manufacturer's table, compiled from experiments, such as that given on next page.

CALCULATED BREAKING LOADS IN TONS, OF STEEL JOISTS, USED AS COLUMNS, WITH ENDS FIXED. THE WORKING LOAD SHOULD NOT EXCEED 1-4TH FOR STATIONARY LOADS, AND 1-6TH FOR MOVING LOADS.

(Taken from Messrs. Dorman, Long & Co.'s Catalogue.)

Reference No.	Section.	Weight per Foot in Lbs.	Length of Column in Feet.											
			7	8	9	10	11	12	13	14	15	16	18	20
			Breaking Weights in Tons.											
B S B 29	20 × 7½	89	628	607	582	563	542	514	488	456	435	404	353	309
„ 28	18 × 7	75	523	508	484	467	445	423	401	375	352	329	282	245
„ 27	16 × 6	62	407	394	369	346	326	301	277	255	230	211	182	155
„ 26	15 × 6	59	395	377	358	333	316	292	266	246	226	208	177	152
„ 24	14 × 6	57	374	356	341	321	300	283	258	237	225	204	174	150
„ 23	14 × 6	46	308	293	278	259	241	224	206	189	171	153	135	115
„ 22	12 × 6	54	367	350	337	321	302	283	262	242	222	206	171	153
„ 21	12 × 6	44	298	286	274	260	245	230	214	196	182	169	143	124
„ 20	12 × 5	32	200	188	174	160	144	131	111	107	99			
„ 17	10 × 5	29	184	173	162	149	136	124	113	100				
„ 16	9 × 7	58	409	397	383	370	356	338	320	304	283	266	232	201
„ 14	8 × 6	35	239	228	218	208	196	183	170	156				
„ 11	7 × 4	16	83	73	63	55	49	44						
„ 10	6 × 5	25	160	151	142	133	124	112	102	93				
„ 6	5 × 3	11	51	43	37	32	28	24						
„ 4	4 × 3	9½	43	37	31	27	23	20						

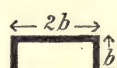
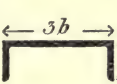
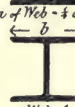
Metal Struts.

The following Table, extracted from "Building Construction," Vol. IV. (Longmans), and compiled from a similar Table given in "Fidler on Practical Strength of Columns," may prove useful.

$\frac{nl}{b}$	Safe stress per square inch of cross section in tons.					
	Cast iron.		Wrought iron.		Mild steel.	
	Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.
10	8·68	8·85	4·00	4·00	5 33	5·34
15	8·41	8·76	3·98	4·00	5·26	5·31
20	8·07	8·65	3·92	3·99	5·20	5·29
25	7·58	8·46	3·88	3·98	5·13	5·24
30	6·98	8·21	3·80	3·95	5·02	5·20
35	6·32	7·91	3·72	3·92	4·90	5·15
40	5·68	7·56	3·64	3·89	4·76	5·09
45	5·02	7·19	3·54	3·86	4·58	5·03
50	4·43	6·82	3·44	3·82	4·40	4·98
55	3·35	6·46	3·31	3·78	4·22	4·92
60	2·92	6·10	3·17	3·73	4·02	4·83
65	2.57	5·75	3.04	3·68	3·80	4·75
70	2·23	5·39	2·90	3·63	3·59	4·67
75	1·96	5·02	2·76	3·55	3·37	4·56
80	1·94	4·68	2·60	3·48	3·15	4·45
85	1·47	4·33	2·46	3·40	2·96	4·35
90	1·56	4·00	2·33	3·32	2·77	4·25
95	1·42	3·66	2·18	3·22	2·56	4·13
100	1·29	3·35	2·03	3·17	2·40	4·00
105	1·17	3·07	1·92	3·08	2·24	3·88
110	1·07	2·80	1·79	3·00	2·08	3·74
115	0·99	2·57	1·67	2·91	1·95	3·61
120	0·93	2·37	1·57	2·82	1·83	3·46
125	0·86	2·19	1·47	2·74	1·71	3·32
130	0·80	2·03	1·39	2·66	1·61	3·21
135	0·75	1·90	1·32	2·58	1·50	3·09
140	0·70	1·78	1·24	2·48	1·42	2·96
145	0·66	1·66	1·17	2·40	1·36	2·85
150	0·61	1·56	1·10	2·32	1·28	2·72
160	0·56	1·40	0·98	2·14	1·13	2·51
170	0·49	1·25	0·88	2·00	1·01	2·32
180	0·43	1·14	0·80	1·84	0·91	2·13
190	0·39	1·03	0·72	1·70	0·83	1·97
200	0·36	0·93	0·66	1·57	0·75	1·83
210	0·32	0·84	0·58	1·46	0·68	1·68
220	0·30	0·77	0·55	1·35	0·62	1·55
230	0·28	0·70	0·50	1·26	0·58	1·44
240	0·25	0·64	0·46	1·18	0·53	1·34
250	0·23	0·59	0·42	1·11	0·49	1·25
260	0·22	0·56	0·40	1·04	0·46	1·16
270	0·20	0·52	0·37	0·97	0·42	1·08
280	0·19	0·49	0·35	0·91	0·39	1·01
290	0·18	0·46	0·32	0·86	0·37	0·96
300	0·17	0·43	0·30	0·80	0·35	0·92

l = length of strut in inches.

b (in inches) and n have the following values for various sections in general use.

Section.	n.	Section	n.	Section.	n.
	3·5		6·0		4·9
	2·5		4·9		3·9
	3·0		4·2		3·8
	4·0		5·9		4·3
	5·1		4·2		4·9

Example.—Find the safe stress for an L steel strut, 3 in. × 3 in. × ½ in., 10 ft. long, the ends being considered rounded or hinged, as in a roof. We have $b = 3$, $n = 4{\cdot}9$, $l = 120$; hence

$$\frac{nl}{b} = 4{\cdot}9\,\frac{120}{3} = 196.$$

From the Table the safe stress per square inch is 0·80 ton nearly. Hence

Safe stress $= 0{\cdot}80 \times (3 \times \frac{1}{2} + 2\frac{1}{2} \times \frac{1}{2}) = 2{\cdot}20$ tons.

Wooden Pillars.

When considering square wooden pillars, Gordon's formula may be considerably simplified. If B W be breaking load in tons per square inch of sectional area, l be length in inches, b be breadth in inches, and C crushing strength of timber in tons per square inch.

$$\text{B W} = \frac{\text{C}}{1 + \left(\cdot 004 \frac{l^2}{b^2}\right)}$$

Kinds of Wood.

	Crushing Strength per sq. Inch in Tons.		Crushing Strength per sq. Inch in Tons.
Beech	3·75	Yellow American Pine	1·75
Red Deal ...	2·75	Pitch Pine ...	3·00
Elm	2·75	Teak	4·00
Mahogany ...	3·66	Greenheart ...	6·00
English Oak ...	3·75		

The following Table, giving the Safe Load in Tons upon Pillars of Red Fir, taken at 1-10th the breaking weight, may be useful :—

Side in Ins.	Length of Pillar in Feet.					
	8	10	12	14	16	18
4	1·2	0·9	0·6	0·5	0·4	0·3
5	2·5	1·9	1·5	1·1	0·9	0·7
6	4·5	3·5	2·7	2·2	1·8	1·5
7	7·1	5·7	4·6	3·7	3·1	2·5
8	10·3	8·6	7·0	5·9	4·9	4·2
9	14·2	12·0	10·2	8·6	7·3	6·2
10	17·6	16·3	13·9	12·0	10·3	8·9
11	23·6	20·9	18·3	15·9	13·9	12·1
12	29·1	26·2	23·3	20·6	18·1	16·0

Wooden pillars should not be loaded with more than 1-10th the breaking weight on account of the great variation in strength of different specimens, and the rapid fatigue under great loads.

CHAPTER IX.

FRAMED CANTILEVERS.

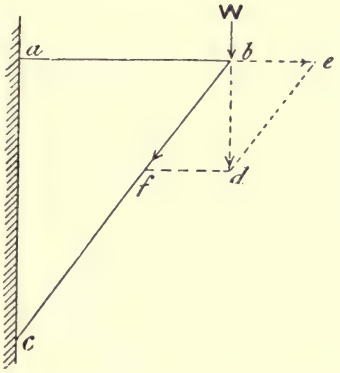
The stresses in the different members of all framed structures can be ascertained by applying to them either the parallelogram or the polygon of forces, or both. The stresses discovered are those of tension and compression only, and it is assumed that they act in straight lines between the pins joining the different members together, and these lines alone are represented on the diagrams appended to this and the succeeding chapters.

The most simple form of framed structure is the framed cantilever, commonly known as the bracket or crane, according to the use to which it is put.

The stresses upon the members of a single bracket (Fig. 71) are readily ascertained by means of the parallelogram of forces. Assuming the bracket to be loaded only at the end, set down vertically below the point of application of the load W a line *b d*, representing W to any scale of weights, the load acting downwards in the direction of the arrow. Round *b d* as a resultant form the parallelogram *b e d f*, with its sides parallel to the members *a b* and *b c* of the bracket; then *b e* will represent the stress in *a b*, and *b f* the stress in *b c*, to the same scale as that to which *b d* was set down. The direction of the stresses is shown by the arrows, *a b* being in tension on account of the pull of *b e* from the wall, and *b f* in compression, on account of the push of *b f* towards the wall.

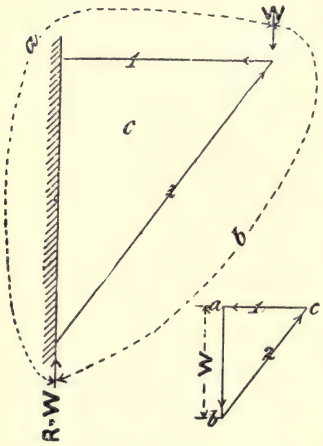
The same result can be arrived at arithmetically with the help of Euclid I. 47, if the lengths of *a b*, *b c*, and *c a* be known (*a b c* and *b e d* being similar triangles), or by means of trigonometry, if either of the angles *a b c* or *b c a* be known. Given neat draughtsmanship, however, the graphic method of determining stresses is as accurate as either of the others, and its rapidity gives it a great advantage in practice. This advantage also can be greatly increased, especially in complicated structures, by the employment of reciprocal diagrams. The application of one of these to the simple case now under consideration is shown in Fig. 72, in which, for simplification, Bow's notation of the spaces is employed. The reaction of the support is known to be equal to W (because the whole weight is borne by the support), and it acts upwards at the foot of the bracket. The *space* lying between R and W, over the bracket, is lettered *a*, that between W and R, under the bracket, *b*. Then the space enclosed by the bracket must be lettered *c*. Great pains must be taken to grasp the fact that *spaces* are now being lettered on the sketch, and that these spaces correspond with similarly lettered *points* on the reciprocal diagram, in the following manner:—Between the spaces *a* and *b* there lies the weight W acting vertically downwards. From the point *a* to the

Fig. 71.



point *b* set to any scale of weights a distance vertically downwards equal to W. Then between the spaces *b* and *a* there lies the reaction R acting vertically upwards, this being represented on the diagram by the line *b a*, drawn vertically upwards. A closed polygon, *a b*, *b a*, is thus formed in the one line between the points *a* and *b*, representing the external forces acting on the bracket, showing that these forces are in equilibrium.

Fig. 72.



In order to complete the diagram, and to ascertain the position of the point to represent the space *c*, as well as to discover the stresses in the two members of the bracket, it is necessary to consider the forces acting at one of the joints. For the sake of convenience it is best to choose the joint at the apex of the bracket. Here, between the spaces *a* and *b*, there lies the load W, already represented by the line *a b*. Between the spaces *a* and *c* there lies the horizontal member of the bracket; from the point *a* in the diagram, therefore, draw *a c* horizontally and of unlimited length. Between the spaces *b* and *c* there lies the inclined member of the bracket; from the point *b* in the diagram draw *b c* parallel to this member till it cuts *a c* in *c*. Then, to the scale to which W was set down in the diagram, *a c* represents the stress in the member lying between the spaces *a* and *c*, and *b c* represents the stress

in the member lying between the spaces *b* and *c*, the letters used reciprocating, and the triangle *a b c* in the diagram forming the triangle of forces about the apex of the bracket.

The stress in 2 is again divided at its foot into R, acting vertically upwards, and a thrust, equal to the stress upon the line 1, acting horizontally. This last has not been shown on the diagram for the sake of clearness.

Not only the value, but the nature also, of these stresses can be shown on the sketch. It is one of the properties of a triangle or polygon of forces of any number of sides, that if it be properly closed and represent in order the forces acting upon a point (such forces being in equilibrium), the direction of these forces will follow in consecutive order. Of the forces at the apex of the bracket, W is known to act vertically downwards from *a* to *b*. The other forces will therefore act, as shown by the arrows on the reciprocal diagram, from *b* to *c* and from *c* to *a*. If these arrows be transferred from the diagram to the sketch, it will be seen that that on the inclined member pushes towards the joint under consideration, while that on the horizontal member pulls away from the joint—in other words, the latent force in the member between *b* and *c*, which meets the stress upon that member, pushes towards the joint, the member being in compression, while the member between *a* and *c* is in tension.

The above lengthy explanation of a simple principle should be most carefully studied and mastered, as upon it depends the unravelling of many a complicated problem.

The stresses upon joints are of as great importance as are those upon the members of a structure, for the strength of the whole is only that of its weakest part. If, as is usual, the weight upon the bracket be carried at the apex as shown, resting upon the extremity of the horizontal

member, to which the inclined member is pinned, the shear on the pin (which generally exposes two shearing areas, each meeting half the shearing stress) is equal to the resultant of the load W and the tensional stress in the horizontal member—or in other words, is equal to the compressed stress in the inclined member.

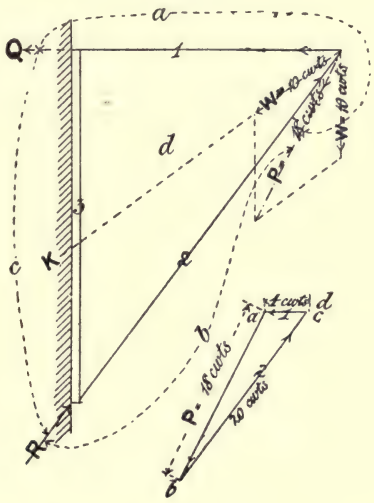
Should the bracket carry a distributed load, half of this may be considered to be borne by the wall direct, and the remaining half to act as an end load, in which case the stresses due to this must be ascertained in the same manner as were those in the last example, substituting $\frac{W}{2}$ for W. Additional to the tensional stress thus found to exist in the horizontal member, however, it must be remembered that it acts as an ordinary beam carrying a distributed load between two supports (viz., the wall and the apex of the bracket), and that it must be proportioned accordingly.

When the cantilever is used as a crane—viz., when a rope or chain passes over a pulley at the apex of the bracket to support a weight—it is not this weight only which goes to form the stresses in the members. The rope passing over the pulley pulls on it in two directions (see Figs 73 and 74), the resultant P being the pressure bearing upon the pulley and causing the stresses. It is this resultant, or the thrust in the member 2, whichever may be the greater, which has to be borne as a shear upon the pin at the apex joint.

The cranes shown in Figs. 73 and 74 are in the form of simple brackets, resting upon eyes let into the wall so that they may rotate freely to any required position. The chain (which is presumed to carry a weight of 10 cwt.) passes over the pulley and down to a winch at the point K. The weight and the winch each pull upon the chain to

the extent of 10 cwt., causing a resultant pressure upon the pulley P, which is equal to 18 cwt. in amount, and the direction of which is shown in the diagrams. In such a crane two cases are possible: it may be supported either by hanging on to the eye at its head, or by resting on the stirrup at its heel. In the latter case the diagram is a simple one, and is given in Fig. 73. We know at once

Fig. 73.



that the external forces acting on the crane are the pressure P, the reaction R at the heel, equal in amount and opposite in direction to the stress on the member 2, and a direct resistance Q at the head, equal to the stress on the member 1. Common sense, a useful factor in the investigation of stresses as in many other matters, tells us that there is no stress in the member 3; and that the stress in 2 is conveyed in its full original force and direction to the

wall by the eye at the foot acting as a very short cantilever—this eye being subject to a vertical end load and a horizontal thrust, respectively the vertical and horizontal components of the stress in 2, into which these can be again resolved on reaching the wall.

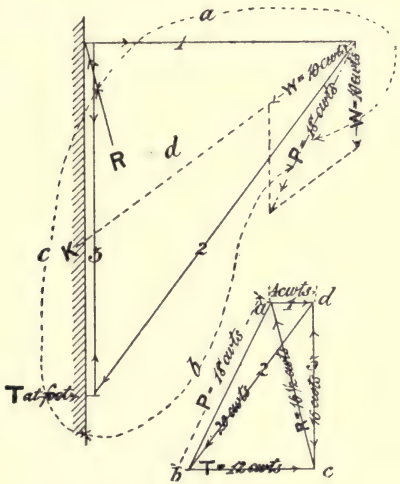
The space between Q and P on the sketch should be lettered *a*, that between P and R lettered *b*, and that between R and Q lettered *c*. The space in the centre of the bracket may be lettered *d*.

Between the spaces *a* and *b* there lies the pressure P of 18 cwt. Lay down, therefore, on the diagram from the point *a* to the point *b* a distance equal to 18 cwt., to a scale of weights to represent P and parallel to its direction. The other external force being as yet unknown, it is impossible to lay it down; but the stresses in the members 1 and 2 can be ascertained by drawing *a d* and *b d* (1 and 2) on the diagram respectively parallel to the members 1 and 2 lying between the spaces *a* and *d*, and *b* and *d*. The stress on 1 is therefore represented by the line 1 on the diagram, which equals 4 cwt. to scale; and the stress on 2 is similarly shown by the line 2, and is equal to 20 cwt. The direction of these stresses is shown by the arrows, for P acting downwards, the arrow was set downwards from *a* to *b* on the diagram, and the other arrows on the diagram follow in rotation round the closed polygon from *b* to *d* and from *d* to *a*. These being transferred to the sketch show the arrow on 2 to point towards the joint under consideration, the member 2 being, therefore, in compression, while the arrow on 1 points from that joint, showing that it is in tension.

There being no stress in the member 3, it is evident that the spaces *d* and *c*, between which it lies, may be represented by one and the same point (lettered *d* and *c*) on the diagram. R, therefore, lying between *b* and *c*, is

equal in amount and direction to *b c* on the diagram, the polygon of forces acting at the foot of the bracket being represented by the lines *d b*, *b c*, *c d*, which last is *nil;* and R may be divided up into vertical and horizontal resistances if required, as explained in the previous case. Similarly the polygon *c a*, *a d*, *d c* represents the polygon

Fig. 74.



of forces about the head of the bracket, *c a* being opposite and equal to *a d*, and representing Q, the resistance of the wall at the head of the bracket.

The case shown in Fig. 74, where the crane hangs at its head, requires somewhat further consideration. The external forces here, besides P, are a horizontal thrust, T, at the foot, and the reaction R, at the head, of which neither the value nor direction is known, save that it is

equal and opposite to the resultant of the stresses in the members 1 and 3.

The lettering may be similar to that of Fig. 73, the space between R and P being lettered *a*, that between P and T being lettered *b*, and that between T and R being known as *c*, the space in the middle of the bracket being *d*.

From *a* to *b* on the diagram a distance may be set down as before, equal in amount and parellel in direction to P, and the triangle *a b d* may be completed, representing the pressure P and the stresses in 1 and 2. So far this is exactly similar to the previous diagram, the stresses, both in amount and direction, in the members 1 and 2 being the same as those then ascertained. For the sake of clearness the arrows relating to this triangle are not again shown.

Both at the head and foot of the bracket, however, there are two, as yet, unknown forces. At the foot the directions of both (the thrust T, and the stress in 3) are known, which is not the case at the head, where the direction as well of the value R remain unknown. By drawing *b c* on the diagram parallel to T, which lies between *b* and *c*, and by drawing *d c* parallel to 3, which lies between *d* and *c*, it is possible to fix the point *c* on the diagram to represent the space *c* on the sketch. The value of T is thus ascertained, by scaling *b c*, to be equal to 12 cwt., while the value of 3 is similarly found, by scaling *c d* on the diagram, to be 16 cwt., all the forces at the foot being represented by the triangle *b c d*.

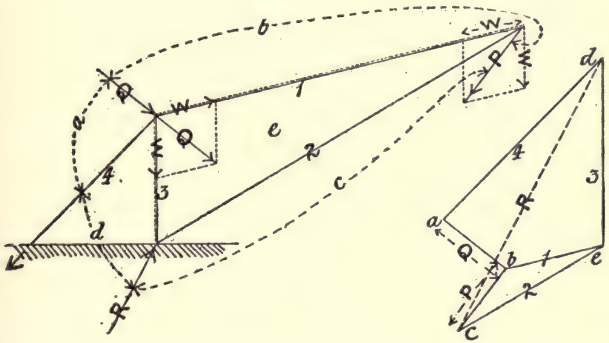
The member 2 is known (from the consideration of the previous case) to be in compression, and therefore thrusts towards the joint at its foot, the arrow pointing downwards. This arrow is transferred to the diagram, pointing from *d* to *b*, the other arrows following round the triangle *d b c* in rotation from *b* to *c* and *c* to *d*. These being

transferred back to the sketch, show T thrusting towards the joint, and 3 in tension pulling away from it.

At the head of the bracket there is, now that the value of 3 has been ascertained, by one unknown force, R ; and this can be found, as it lies between the spaces *a* and *c*, by joining *a c* on the diagram. Its direction is parallel to *a c*, and its value $16\frac{1}{2}$ cwt.

The forces acting at the head of the bracket are now represented by the sides of the triangle *a d c*. The

Fig. 75.

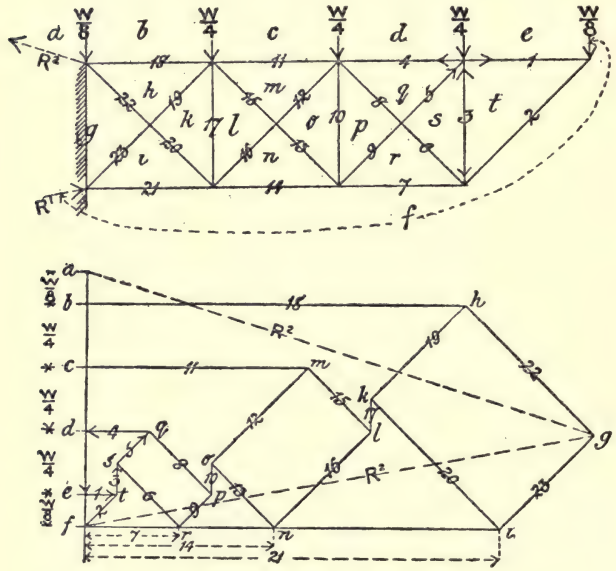


member 3, lying between *d* and *c*, has just previously been found to be in tension, and therefore pulls away from the joint. This arrow, therefore, points downwards from *d* to *c* on the diagram (*i.e.*, in the opposite direction to the arrow which denoted that the member 3 pulled away from the previously considered joint at the heel), and the other arrows, following in rotation from *c* to *a* and *a* to *d*, show, when transferred to the sketch, that R acts upwards, and that 1 pulls away from the joint and is in tension.

Fig. 75 exemplifies the case of a derrick crane, taken when the stresses are greatest, that is, when the gib is so

placed as to be in line on plan with one of the back stays. The weight W is supposed to be suspended at the end of the chain shown by dotted lines, so that a resultant pressure P, ascertained as already described, is found to act at the junction of members 1 and 2, while a similarly

Fig. 76.



ascertained resultant pressure Q acts at the junction of members 1, 3 and 4. This force Q may be assumed to act externally to the structure, it being an axiom that a force may be assumed to act at any point in the line of its direction, and so may be considered to separate the spaces *a* and *b*, as P separates the spaces *b* and *c*. Between the spaces *c* and *d* will lie a reaction R, whose value and

direction are at present unknown to resist the combined thrusts upon members 2 and 3.

Further explanation is hardly necessary, as the method of forming the stress diagram has been already described, the resultant Q, acting between spaces *a* and *b* on the sketch, being represented by the line Q, equal to it in value and parallel to it in direction, set down from point *a* to point *b* on the diagram; and the distance P being similarly set down parallel and equal to the resultant P from point *b* to point *c*. The triangle *b c e* is then completed, representing the resultant P and the stresses in the members 2 and 1; and then the line *e d* is drawn parallel to member 3 and the line *a d* parallel to the member 4, completing the polygon *a b e d a* which represents the resultant force Q and the stresses in the members 1, 3 and 4. The direction and value of the reaction R are found by joining the points *c* and *d* on the diagram, as it separates the spaces *c* and *d* on the sketch.

The next case considered (Fig. 76) is that of a cross braced lattice cantilever supporting an evenly distributed load on the top flange. Of the total load W, one-fourth lies on each of the members 18, 11, 4 and 1, and is carried by them to their points of support. Thus $\frac{W}{8}$, being one-half of the load lying on member 18, may be supposed to act as shown on sketch between the spaces *a* and *b*; while between the spaces *b* and *c* there will act a load equal to $\frac{W}{4}$, this being the sum of half the loads lying on 18 and 11. Similarly, $\frac{W}{4}$ acts downwards between *c* and *d*, $\frac{W}{4}$ again between *d* and *e*, and $\frac{W}{8}$ between *e* and *f*.

These loads, acting vertically downwards, are represented

on the diagram by lines drawn vertically downwards to a scale of weights, $\frac{W}{8}$ from point a to point b (representing $\frac{W}{8}$ lying on the sketch between space a and space b), $\frac{W}{4}$ from point b to point c, and so on.

It will be noticed that at the outermost extremity of the cantilever the load $\frac{W}{8}$ and the members 2 and 1 are dispositioned much like the load W and the members 2 and 1 in Fig. 72. Similarly to what was done in that case, it will be seen that in Fig. 76, $\frac{W}{8}$ has been laid down vertically from e to f on the diagram, and that lines 2 and 1, meeting at t, have been drawn parallel to the lines 2 and 1 on the sketch. The length of the line 2 on the diagram represents, to the scale to which $\frac{W}{8}$ was laid down, the stress in the member 2 on the sketch; and in the same way the length of the line 1 represents the stress in the member 1.

The point t on the diagram has now been found, representing the space t on the sketch.

A new difficulty now occurs, however, for of the members 1, 3, 5 and 4 which meet at the head of the member 3, the stress in member 1 alone is known, and the polygon of forces cannot be completed so long as the stresses in more than two remain unascertained; and there is a similar condition of affairs at the foot of the member 3, where 3, 2, 7 and 6 meet, the stress in 2 being the only one which has yet been found. Common sense must now be brought into requisition, and it tells us that, with theoretically correct jointing, such as may be assumed

with good English workmanship, as soon as the cantilever has taken its bearing, half the load upon the head of 3 will act there to cause stress in 4 and 5, and half be conveyed by 3 to its foot to there cause stress in 6 and 7. By this reasoning, 3 may be assumed to be in compression to the extent of half of $\frac{W}{4}$, which is $\frac{W}{8}$. Being in compression it will thrust outwards towards each of its extremities, and the arrows may thus be pointed.

The polygons both at head and foot of 3 can now be completed. Taking the one at its head, $\frac{W}{4}$ acts downwards between spaces *d* and *e*, and its representative line has been drawn downwards on sketch from point *d* to point *e*. The sequence of the lines in the polygon being thus given, it is seen that line 1, acting from *e* to *t* on diagram, represents the stress in member 1 on the sketch, the arrows showing this to be in tension. Then between spaces *t* and *s* on the sketch, the line 3 is seen to act *upwards* (compression) towards its head, the value of its compression being known to be $\frac{W}{8}$. On the diagram, therefore, line 3 must be drawn *upwards* from point *t* to point *s*, so as to equal $\frac{W}{8}$ to the scale of loads. The polygon is then completed by drawing line 5 from point *s* on the diagram parallel to line 5 on the sketch, and line 4 from point *d* on the diagram parallel to line 4 on the sketch, meeting at point *q*, the sequence of the direction arrows, in continuation of those already laid down, being from *s* to *q* and from *q* to *d*. On these arrows being transferred to the sketch round the joint under consideration it is seen that member 5 is in compression, while 4 is in tension.

The remainder of the diagram need not be particularly described, as no fresh difficulty arises, the values and directions of the reactions R^1 and R^2 being found by joining points $f\ g$ and $a\ g$ on the diagram, as R^1 separates space f from space g on the sketch, and R^2 separates space a from space g.

CHAPTER X.

FRAMED BEAMS AND GIRDERS.

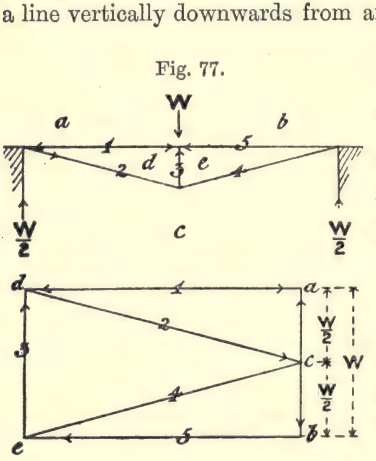
The stresses on all framed structures can be ascertained mathematically by solving triangles, parallelograms, and polygons of forces; but as all these, round all the joints in a structure, can be included in one diagram of the kind used in the last chapter, and as such a diagram requires only accurate draughtsmanship for its solution in the place of a considerable acquaintance with mathematics, it is now almost universally employed; although on structures of magnitude and importance it is best to calculate the stresses by both methods, that the one may check the other. In this work, however, the diagram will be adhered to.

The simplest example of a framed beam is the trussed traveller (see Fig. 77). The greatest stresses upon the members are found when the weight which it has to carry is over its centre; for if the weight be in any other position a portion of it only would be carried by the strut in the centre (acting as an abutment), the remainder being carried directly by one of the main supports. With the weight in the centre, the reaction of each support will equal $\frac{W}{2}$. Letter the spaces as shown in Fig. 77, taking the spaces outside the structure first, and then those inside, in rotation as the sun moves. The space between the re-action of the left-hand abutment and W is thus

known as a, that between W and the other reaction as b, that between the two reactions as c, that between the members 1, 2, and 3 as d, and that between the members 3, 4, and 5 as e.

Between the spaces a and b there lies the load W, acting vertically downwards. Set to a scale of weights a line vertically downwards from any point a to a point b, to represent W. Between the spaces b and c, $\frac{W}{2}$ acts vertically upwards. Set up vertically from b to c a distance to represent $\frac{W}{2}$; similarly set up from c to a a distance to represent $\frac{W}{2}$ acting upwards between the spaces c and a. The line $a\,b\,c\,a$ on the diagram now forms a closed polygon, which represents the forces W, $\frac{W}{2}$ and $\frac{W}{2}$ acting externally upon the structure, and which are themselves in equilibrium.

Fig. 77.



In order to ascertain the stresses in the members, it is necessary to form, on the diagram, polygons of the forces which act at each of the joints in succession, and it is usual to commence with the joint over the left-hand abutment. Here, besides the reaction $\frac{W}{2}$ upwards between c and a, there act the stresses in the members 1 and 2,

As the line 1 lies between the spaces *a* and *d*, draw a line parallel to it on the diagram from *a* till it is cut by another from *c* drawn parallel to the line 2, which lies between *c* and *d*, these new lines on the diagram meeting at the point *d*. Then *a d* on the diagram represents to scale the stress on the member lying between the spaces *a* and *d*, and *c d* represents the stress upon the member lying between *c* and *d*; the lines $\frac{W}{2}$, 1, and 2 forming a close polygon in the diagram. $\frac{W}{2}$ is known to act upwards, and the other forces, following in rotation, necessarily act as shown by the arrows from *a* to *d* and from *d* to *c*. These arrows, on being transferred to the sketch, show 1 pushing towards and 2 pulling from the joint—in other words, 1 is in compression and 2 in tension.

Hitherto three forces only have acted on any one point; but directly under the load four forces will be found—viz., W and the stresses 1, 3 and 5. The values of two of these only are unknown, and the others can therefore be found, their directions being known, and it being possible to close the polygon about the joint by drawing *d e* on the diagram parallel to the line 3 lying between *d* and *e* on the sketch, and meeting it by *b e* drawn parallel to the line 5 lying between *b* and *e* on the sketch. The line 1 is known to be in compression, and therefore acts towards this joint, just as it was shown to do towards the joint at its other extremity, and its direction may therefore be represented by an arrow from *d* to *a* on the diagram. W acts downwards from *a* to *b*, maintaining the rotation, which is kept up by 5 acting from *b* to *e*, and 3 acting from *e* to *d*. This closes the polygon, 1, W, 5 and 3 on the diagram, representing the stresses in 1, W, 5 and 3

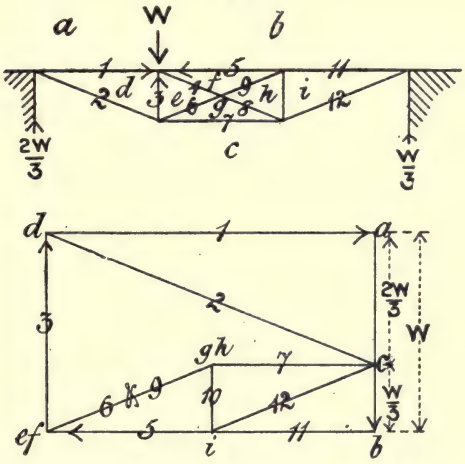
respectively in the sketch, and the direction arrows, when transferred, show that 5 and 3 are both in compression, acting towards the joint. It will be noticed that the stress in 3, which is directly under, and has its line of action coincident with that of the load, is equal to W. The stress in 4 can be ascertained by joining *c e* on the diagram, and if the working has been correct *c e* should be parallel to the line 4 (which lies between the spaces *c* and *e*) on the sketch. The direction in which it acts can be found by following the arrows round either of the polygons of forces of which *c e* forms a part—viz., either of those representing the forces acting at one of the extremities of the line 4. These arrows have not been shown, to avoid confusion, 4 being in tension.

A larger trussed traveller, with two struts and cross braces, is shown in Fig. 78. The greatest stresses upon the members are evidently produced when the weight is directly over one of the struts; but it must be carefully borne in mind in designing such a structure that the weight, which is a travelling weight, will sometimes be over one strut and sometimes over the other, while the members 1, 5 and 11 must be made sufficiently strong to carry it as a load placed centrally upon either of them.

The solution is very similar to that of the previous example until the joint under the load is reached, save that the reactions are not equal. The stresses in the members 1 and 2 can be ascertained as previously explained; but at the joint under the load the forces which act are five in number—viz., the load W, and the stresses in the members 5, 4, 3 and 1. Of these, three are unknown, so that without an assumption the polygon cannot be completed, nor can the stresses upon these members be determined. By calling in the assistance of common sense, however, in this, as in most cases, an assumption is rendered

possible which will permit of the required solution being arrived at. Bearing in mind what occurred during the solution of the previous example, we may fairly assume here that the member 3, being directly under and coincident in direction with the weight, is placed in compression to the extent of that weight. Set down on the diagram, therefore, a vertical line *d e* to represent the stress in the

Fig 78.



member 3, lying between the spaces *d* and *e*, equal in value to W. The member 1 being in compression acts towards the joint, this being represented by the arrow from *d* to *a* on the diagram. The member 3 we know to be also in compression, so that it acts upwards towards the joint; and, therefore, the distance *d e* must be set downwards so that the arrow may act upwards from *e* to *d* to preserve the continuity of direction of the arrows from *e* to *d*, and *d* to *a*. The point *e* is thus absolutely fixed, and

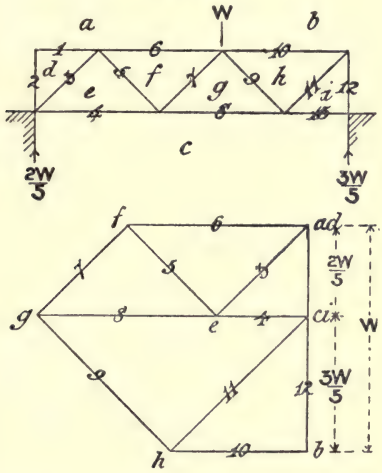
it becomes possible to complete the polygon of forces by drawing *e f* from *e* and *b f* from *b*, parallel to the lines 4 and 5 (lying between the spaces *e* and *f* and *b* and *f*, respectively) till they meet in the point *f*—which in this instance occurs at *e*, this being from the construction in a horizontal line with *b*, thus showing that there is no stress upon the member 4. The direction arrows pertaining to the polygon *a b f e d a* are the only ones shown on this diagram, to avoid confusion, and these show that all the members attached to the joint under the load are in compression. The completion of the diagram is now easy upon the lines already explained, there being no stress on the member 8—the only polygon in any way out of the common being *b i h f b*, representing the stresses in the members 11, 10, 9 and 5, where *b i* and *f b* lie in the same straight line but in opposite directions.

Having so far cleared the way, the consideration of framed girders of any ordinary type should become a comparatively easy matter, and no difficulty whatever should be found in determining the stresses upon the members of the Warren girder shown in Fig. 79, so long as it is subject to only one load at any point. With the load as shown there would be no stress upon either of the members 1, 2 and 13, while the member 12 would be in compression to the extent of the reaction of the abutment under it. The arrows pertaining to the forces acting at the various joints are not shown, but the force polygons can be readily followed, the members 6, 10, 3, 7, 9 and 12 being in compression, and all the others which are subject to any stress at all in tension.

Girders of the form shown in Fig. 79 are not often employed, but they are occasionally of great value, especially when a bridge has to be thrown where there are two good natural abutments at different levels, the

members 12 and 13 being dispensed with, 10 and 11 hanging on the abutment, as shown in Fig. 80, which is shown carrying a distributed load on the top flange. The members 1, 6 and 10 act as a continuous girder, although, as a rule, in practice they are calculated as three separate supported girders, each carrying a distributed load, in addition to the stresses brought upon it as part of the framed structure. In either case, half of

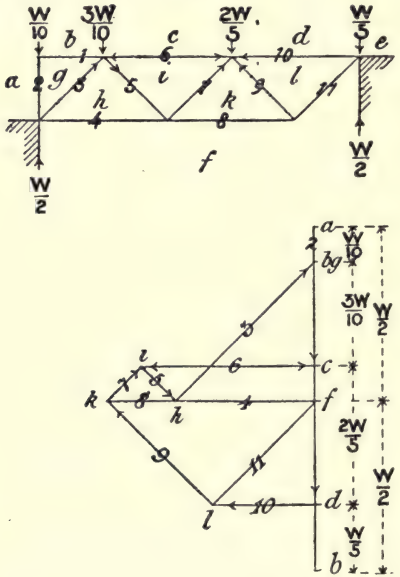
Fig. 79.



the load on 1 is carried by each of its abutments, and so with 6 and 10, making the loads on these supports $\frac{W}{10}$, $\frac{3W}{10}$, $\frac{2W}{5}$, and $\frac{W}{5}$ respectively as shown, in proportions of the total load W. The load being distributed, the reaction of each abutment is $\frac{W}{2}$, and the setting-down of

the polygon of external forces is easy, $\frac{W}{10}$, $\frac{3\,W}{10}$, $\frac{2\,W}{5}$, and $\frac{W}{5}$ acting downwards, and each reaction of $\frac{W}{2}$ acting upwards, forming the closed polygon *a b c d e f a*. The

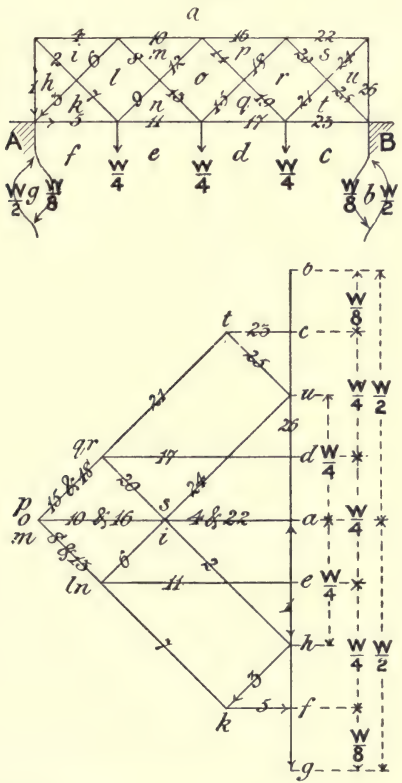
Fig. 80.



polygon to represent $\frac{W}{10}$ and the stress upon the members 1 and 2 is readily found to be *a b g*—all lying in the one line *a b* on account of *b* and *g* coinciding, there being no stress upon 1. Everything else is found as usual but the polygon *b c i h g*, which represents the forces acting at

the joint under $\frac{3\,W}{10}$, is worth notice, on account of the crossing of its boundary lines.

Fig. 81.



In the previously considered cases it would have been possible by inspection to discriminate between the members

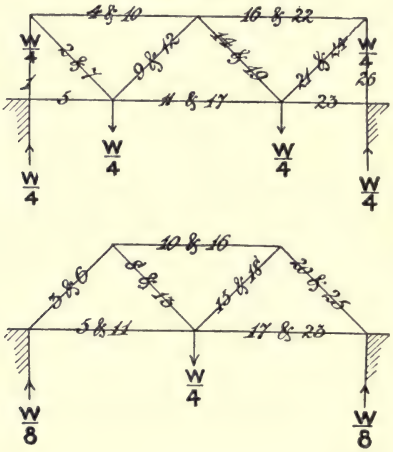
in compression and those in tension; but in such instances as that given in Fig. 80 this is by no means easy, and the arrows should be consulted. They are shown for the polygons of the forces acting about the joints under $\frac{3\,W}{10}$ and $\frac{2\,W}{5}$, and denote that the members 2, 6, 10, 3, 7 and 9 are in compression, while all the rest are in tension, except 1, which bears no other stress than that which comes upon it as a beam bearing the distributed load of $\frac{W}{5}$ to its two supports.

Another common form of framed girder is that shown in Fig. 81, generally known as the "trellis girder." It is assumed in Fig. 81 that the load is distributed and lies entirely upon the bottom flange, as is very commonly the case; and fresh difficulties are met with at once. It is found that at each abutment there act two forces: a small proportion of the load, in this case $\frac{W}{8}$, acting vertically downwards, and the reaction $\frac{W}{2}$ acting vertically upwards. In order to enable the space between these two forces to be lettered, their direction lines have to be looped, as shown. This done, the lettering can be proceeded with as usual, and the polygon of external forces can be readily laid down, commencing with $\frac{W}{2}$ acting upwards from a to b, to represent the reaction which acts upwards between the spaces a and b, proceeding with $\frac{W}{8}$ downwards from b to c, $\frac{W}{4}$ downwards from c to d, and so on, and closing with $\frac{W}{2}$ upwards from g to a, to represent the other reaction between the spaces g and a.

At the abutment A, the two external forces, $\frac{W}{8}$ downwards and $\frac{W}{2}$ upwards, are now known; but the three other forces, viz., the stresses upon the three members 1, 3, 5, are all unknown. It is therefore impossible to complete a polygon of the forces acting at the abutment A, without arriving first at the stress on one of these members by logical reasoning. The readiest way to do this is to dissect the trellis girder which we are considering into the two Warren girders of which it is made up (see Fig. 82), disregarding the small proportions of the load which lie directly on the abutments, and which, consequently, do not go to cause stresses upon the members of the structure. It will be seen at once, by inspection, that the members 1 and 26 are strained only to the extent of $\frac{W}{4}$, and that they are in compression. The stresses in the whole girder may be solved by solving those in the members of these component girders, adding similar and subtracting dissimilar stresses on the same member wherever they occur; but it is much easier to combine the diagrams in one, as shown in Fig. 81. A little practice will make it possible to dispense with the dissection shown in Fig. 82, and to determine the stresses in the members 1 and 26 by means of a mental dissection of the parts only. Having thus determined the stresses in these members, the rest is easy; $\frac{W}{8}$ acting downwards from f to g, and $\frac{W}{2}$ upwards from g to a. $\frac{W}{4}$ (which represents the compressional stress in the member lying between the spaces a and h, and which member, therefore, presses towards the joint), must act downwards from

a to h, to preserve the continuity of direction of the lines in the force polygon. This fixes the point h, and the lines $h\,k$, $k\,f$ follow as a matter of course, determining the stresses upon the members 3 and 5, and completing the polygon $f\,g\,a\,h\,k$. The direction arrows referring to this polygon are the only ones shown; but all others can be ascertained as usual, the members 1, 4, 10, 16, 22,

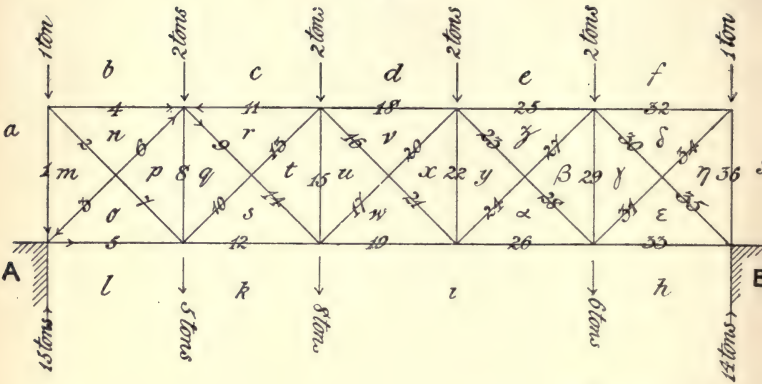
Fig. 82.

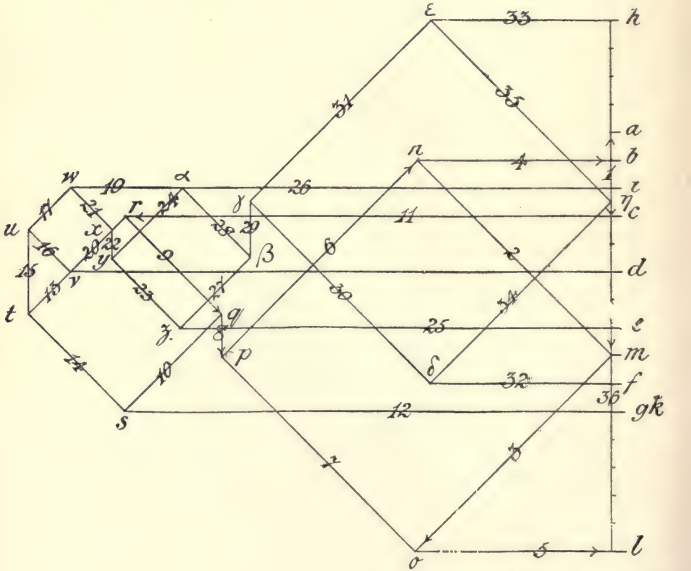


26, 3, 6, 20 and 25 being in compression, and all the rest in tension except 9, 12, 14 and 19, in which equal and opposite stresses meet and neutralise. Theoretically these members might be omitted, but practically it would be unwise to leave them out, as any accidental or temporary inequality of the loading would bring them into use at once, and in any case they are of value in stiffening the structure.

Lattice girders are, perhaps, in more common use than

Fig. 83.





[*To face page* 109.

either Warren or trellis girders, and present difficulties of their own to the solution of the stresses. An example is given in Fig. 83, loaded, as it might be, with an evenly distributed wall of half a ton per foot run on the top flange, and by loads of five, eight and six tons respectively at different points on the bottom flange; these loads being brought upon it by cross girders. The diagram produced is a complicated one; but it only requires care to enable anyone to follow it through. The external loads are set vertically downwards and upwards in succession as usual, till the polygon *a b c d e f g h i k l* is complete, the reactions having been ascertained either arithmetically or by means of the polar diagram. Of the forces at the abutment A, only the reaction (15 tons) is known, the stresses in the three members 1, 3 and 5 being unknown. It requires but little consideration, however, to determine the stress in 1. Of the reaction of 15 tons, 1 ton is met at once by the load on the top flange over the support. The member 1 is therefore put into direct compression to the extent of this 1 ton. Now it may be fairly assumed, as the girder is symmetrically framed, that each of the loads acts half at the top and half at the bottom of the vertical member to which it is attached. Similarly, of the remaining 14 tons of the reaction at A, it may be assumed that 7 tons acts at the foot of the girder, and that the other 7 tons is conveyed by the member 1 to the top flange, putting this member in compression to the extent of 7 tons. It has also been found to be in compression to the extent of 1 ton, which it directly conveys, so that it is totally compressed to the extent of 8 tons. This compression acts towards the joint, and as the reaction between *l* and *a* acts upwards, in order to preserve the continuity of direction of the lines of the polygon of the forces about the abutment, the line *l a*

must be set vertically upwards and the line *a m* vertically downwards. The polygon *a m o l a*, to represent the reaction of 15 tons and the stresses in the members 1, 3 and 5, is now readily completed, the arrows showing that 3 is in compression and 5 in tension. The diagram can be easily continued till points have been found to represent the spaces *n*, *p* and *o*, by which time it will have been discovered that the members 4 and 6 are in compression, and 2 and 7 in tension. At the head of the member 8, however, three unknown forces are met with in the stresses on 8, 9 and 11. The load at the head of 8 is, however, half carried by the framing at its head, and half by that at its foot—in other words, the member 8 conveys one ton of the two tons resting on its head to its foot, and so is put into compression to the extent of one ton. Similarly, in conveying to its head half of the load of 5 tons hanging from its foot, it is put into tension to the extent of $2\frac{1}{2}$ tons. The total stress, therefore, to which the member 8 is subject is $2\frac{1}{2}$ tons $-$ 1 ton; or $1\frac{1}{2}$ tons (compression and tension being respectively designated by the algebraical signs $-$ and $+$, as is often done for convenience).

Similar reasoning will give the stress upon the member 15 as $+$ 3 tons, that in 22 as $-$ 1 ton, and that in 29 as $+2$ tons. These being found, the remainder is easy work. The arrows of direction referring to the members about the head of 8 are shown; but, to avoid confusion, the remainder are omitted: 1, 4, 11, 18, 25, 32, 36, 3, 6, 10, 13, 16, 21, 23, 28, 30, 35 and 22 being in compression, and all the other members in tension.

It must be carefully noted that the assumption that half of each of the supported loads is carried at the head and half at the foot of the vertical member to which it is attached can only be made when the construction is

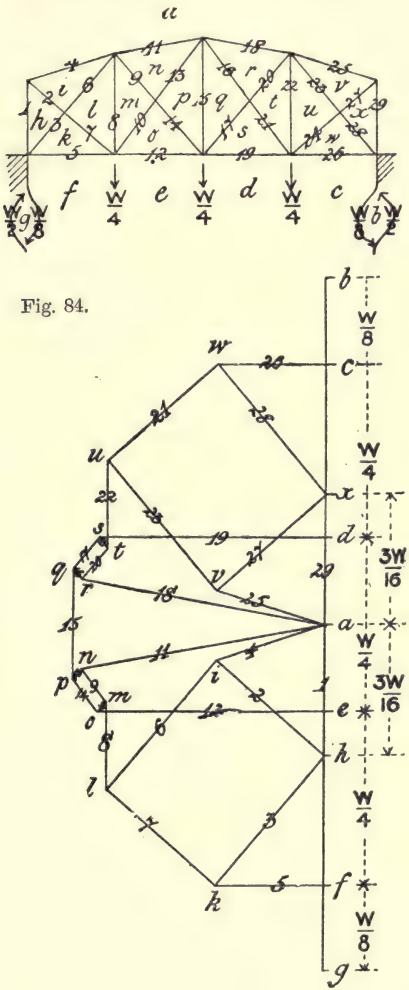


Fig. 84.

sufficiently good for the joints to be taken to be perfect. This may generally be considered to be the case in iron structures, provided that they are put together under efficient supervision.

This assumption also fails if the structure be not symmetrically framed at the heads and feet of the vertical members, as in the bow-string lattice girder shown in Fig. 84, although by that assumption a fair approximation to the truth can be arrived at. By studying the diagram it will be seen that the stresses in the members 1, 8, 22 and 29 have been thus assumed, with the result of discovering the stress upon the member 15 to be one-sixth more than half the load carried at its foot. Had the upper flange been more nearly horizontal, the error would have been less; but the stress upon some of the struts would not have been lessened to so great an extent. The problem is one of a class known as indeterminate—*i.e.*, impossible of accurate determination; but for all that, girders of this form are frequently employed, it being possible to arrive at the stresses upon the members with a sufficient approximation to accuracy for practical purposes, while the æsthetic effect is good; and the stress upon some of the principal struts is less than would be the case were the mean depth maintained throughout. The nearest approximation to the correct stresses is perhaps obtained by making separate diagrams for each system of braces, and averaging the results; but absolute correctness cannot be secured.

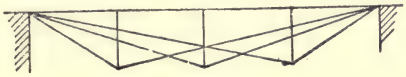
A form used to a considerable extent in bridges abroad, where timber for the struts and hemp or wire ropes for the ties have to be relied on, is shown in Fig. 84. Several modifications are possible; but in any case, to ascertain the stresses on the members, it is necessary to

dissect the framing, treating each simple truss as shown in the example of a trussed traveller given in Fig. 77.

For the common suspension girder another assumption is necessary before the stresses can be ascertained; and the determining of the stresses and the design of the girder must go hand in hand. It is evident (see Fig. 86) that the members 4, 6, 8 and 10 are each in tension to the full extent of the loads suspended from their feet, while the loads resting on the abutments have no effect upon the structure whatever.

In order to proceed, it is necessary either to assume that the central member 7 is horizontal, or to assume the inclination of the two extreme members 3 and 11.

Fig. 85.

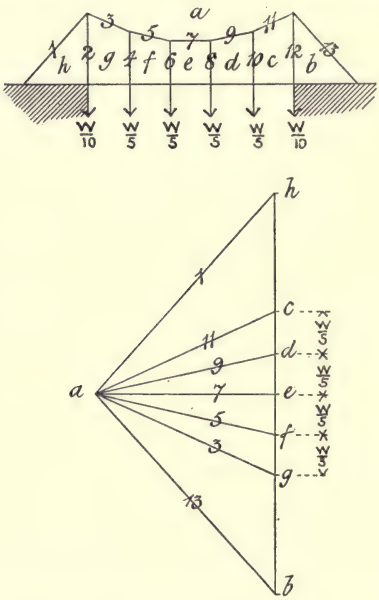


It is also necessary that the stress in 7 should be assumed as well as its direction, if that course be chosen, as its direction alone will not fix the point *a* in the diagram. Join *a c*, *a d*, *a e*, *a f* and *a g*, and draw the members which are not yet determined parallel to their representative lines respectively. By this means both the stresses upon the suspending chain and also its curve are determined, and the completion of the diagram gives the stresses on the vertical supports 2 and 12, and on the ties 1 and 13, which last have to be anchored down by heavy weights, sufficient in amount to resist their pull.

The application of what has gone before to other cases, which may vary to an infinite extent, must now be left to the student's ingenuity. Once the principle

is thoroughly grasped, no problem should prove too difficult of accomplishment, save the indeterminate cases, when approximate results only can be hoped

Fig. 86.



for. Travelling loads do not come, as a rule, within the province of the architect to meet and will not be considered in this work.

CHAPTER XI.

METHOD OF DESIGNING A STEEL LATTICE GIRDER.

By way of a simple example of heavy construction in iron, it has been assumed that a load of 3 tons per foot run, evenly distributed and inclusive of the weight of the girder, has to be carried over a theoretic span of 60 ft., the clear span being 58 ft. The lattice form of girder has been selected, 5 ft. deep, with vertical bars at 6 ft. centres, dividing the span into ten equal bays.

The stresses in the various members can be ascertained as has been already described, the diagram appearing as in Fig. 87, in which the lettering of all the internal spaces, as also the direction arrows showing the nature of the stresses upon the bars (whether compression or tension), have been omitted for the sake of clearness, as the scale is necessarily small; while only half of the stress diagram has been drawn, the other half being similar.

At the present time, mild steel would almost certainly be the material employed, allowing a safe working load of 6½ tons * per square inch of sectional area, whether in compression or tension; and, calculating upon this

* Note.—For British steel, failing under gradually applied tension at between 28 and 32 tons per square inch with an elongation of 20 per cent. on the 8-inch test piece, a safe working load of 7½ tons per square inch may be allowed, but for ordinary Belgian steel 6½ tons per square inch is the proper allowance.

TABLE OF STRESSES in the members of the Girder shown in Fig. 87, together with the sectional area of steel needed to resist them.

Number indicating Member.	Stress in Tons.	Sectional area required in inches at 6½ tons per sq. in.	How built up.	Gross sectional area provided in inches.
1	− 49½	7⅞	Two 4″ + 4″ × ½ L irons, one 30″ × ¾″ gusset, and one plate 16″ × ⅜″	42½
2	− 48½	7⅞	Two 6″ + 6″ × ½″ L irons, and one plate 16″ × ¾″	24
3 and 7	+ 63	9⅞	Two bars 7½″ × ¾″	11¼
4 and 6	− 63			
5	+ 48½	As No. 2	Same as No. 2	24
8, 15, 22, 29 & 36	− 9	1½	Two bars 6″ × ½″	6
9	− 134	20⅝	Two 6″ + 6″ × ½″ L irons, and one plate 16″ × ¾″	24
10 & 14	+ 48½	7⅞	Two bars 7″ × ¾″	10½
11 & 13	− 48½			
12	+ 134	As No. 9	Same as No. 9	24
16	− 200	25¾	Two 6″ + 6″ × ½″ L irons, and two plates 16″ × ¾″ each	36
17 & 21	+ 35	5½	Two bars 7″ × ½″	7
18 & 20	− 35			
19	+ 200	As No. 16	Same as No. 1	36
23	− 243	34½	Two 6″ + 6″ × ½″ L irons, and three plates 16″ × ¾″	48
24 & 28	+ 21	3¼	Two bars 7″ × ½″	7
25 & 27	− 21			
26	+ 242	As No. 23	Same as No. 23	
30	− 265	40¾	Two 6″ + 6″ × ½″ L irons, and three plates 16 × ¾″	48
31 & 35	+ 7½	1¼	Two bars 4″ × ½″	4
32 & 34	− 7½			
33	+ 265	As No. 30	Same as No. 30	48

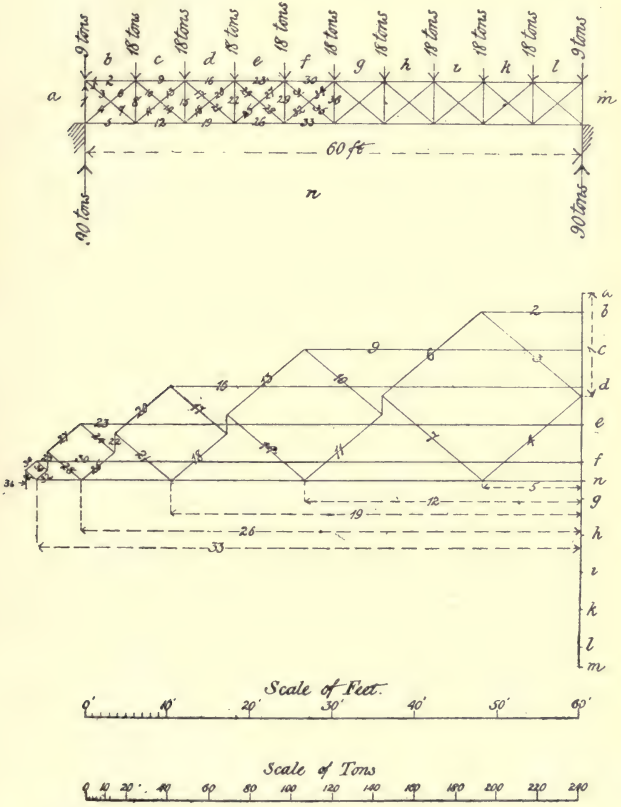
NOTE.—The sign + signifies "Tension," and the sign − signifies "Compression."

assumption, it is well to arrange the data obtained in tabular form as in the first three columns above, the values of the stresses being scaled from the diagram.

It will be noticed that members 30 and 33 are subject to

the greatest flange stresses, in either case 265 tons, which can be safely met by a sectional area of $40\frac{3}{4}$ square inches

Fig. 87.



of metal, allowing a safe load of $6\frac{1}{2}$ tons per square inch, as is usually done for mild steel. This, however, must

be the nett sectional area after deducting rivet holes, for we are now dealing with steel and not with wrought iron, and in this metal $6\frac{1}{2}$ tons per square inch is the safe allowance for compression and tension alike, and while both flanges are built up similarly as a matter of work-shop convenience and practice, it is the tension flange which must be calculated as requiring most metal. It will in this instance be found upon trial that the nearest approach to the required area is obtained by building up the flange with two 6 in. + 6 in. × $\frac{1}{2}$ in. angle-irons, and three plates 16 in. × $\frac{3}{4}$ in. each, giving a gross sectional area of 48 in. With plates of this thickness it would be necessary to use $\frac{7}{8}$ in. diameter rivets. There would have to be deducted from the gross area of flange, therefore, two rivet holes $\frac{7}{8}$ in. × $2\frac{1}{4}$ in. (through three $\frac{3}{4}$ in. plates), and four rivet holes $\frac{7}{8}$ in. × $\frac{1}{2}$ in. (through the angle-irons), this being the greatest possible amount of weakening at any one sectional area, and being equal in all to $5\frac{1}{16}$ in. The nett area is therefore 48 in. − $5\frac{1}{16}$ in. = $42\frac{5}{16}$ sq. in., as against $40\frac{3}{4}$ sq. in. required.

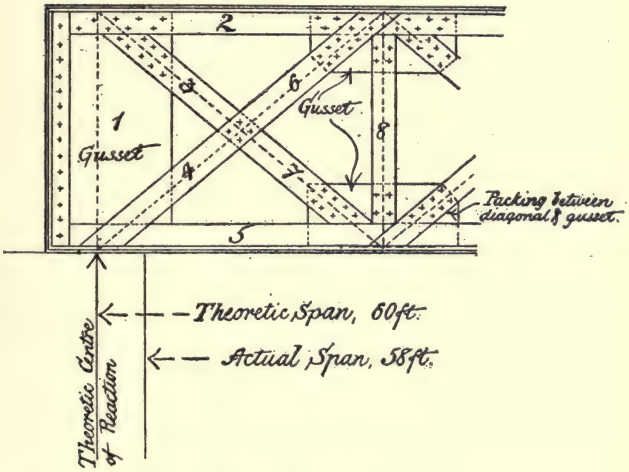
The working sectional areas of members 23 and 26, 16 and 19, 9 and 12, 2 and 5, are similarly determined, and while there is no necessity to repeat the process here, the work has been done (as it must be done in practice), and the result is given in the table. In this connection it must be noted that, having decided to use 16 in. × $\frac{3}{4}$ in. plates, these cannot be pared down in any way—that, if only a very little more metal be required than is contained in two such plates, three must be used, fractional parts of plates being impossible. An instance of this is to be seen in members 23 and 26, where, but for the necessary deduction for rivet holes, two plates would have sufficed.

As the girder is a long one, there would be joints in the

flange plates, and cover plating would be necessary, but as the subject has already been considered in Chapter VII. it is not intended to treat it again here.

We may consequently pass to the consideration of the diagonal braces, of which 3, 7, 4 and 6 bear the greatest stresses, each requiring to have a nett sectional area of at least $9\frac{7}{8}$ in., which, after allowing for two $\frac{7}{8}$ in. rivet holes

Fig. 88.



through each bar, can be obtained by providing two $7\frac{1}{2}$ in. $\times$ $\frac{3}{4}$ in. bars, having a gross sectional area of $11\frac{1}{4}$ sq. in. As the compressional bars 4 and 6 are supported at short intervals there is no necessity to calculate them as columns, or to make special allowance, except to provide against buckling by the construction, so far as is possible; and, with the sizes of the members so far ascertained, the construction is still "tricky."

In Fig. 88 an elevation is given of one end of the girder, showing how it is built up. One principal object to be kept in view is to keep the theoretic centres of stresses (shown in dotted lines) so that they meet properly at the various junctions, and pass along the stress centres of the various members. Thus the theoretic centre of reaction must meet the centre lines of the diagonals 4 and 3, where they in turn meet the flanges 5 and 2; and a similar careful alignment is necessary at foot and head of the vertical member 8. The former of these considerations largely determines the construction of member 1, which is generally built just as is the end of a plate girder (see Fig. 60 for comparison), a gusset taking the place of the web plate.

Similar, but smaller, gussets are inserted between the angle-irons at the heads and feet of the verticals 8, 15, 22, 29 and 36, and it is to these that the tensional diagonals, such as 3 and 7, are attached, being separated (for they are paired, clasping the gussets) by packing pieces where they are themselves crossed and clasped by the compressional diagonals 4 and 6. The compressional diagonals are kept outermost, as a provision against buckling, and so at their extremities clasp both gussets and angle-irons, packing pieces having to be inserted between them and the former.

The size of the gussets is determined by the number of rivets necessary for attachment of the diagonals. Using $\frac{7}{8}$ in. diameter steel rivets, each ·60 square inch in sectional area, and made of metal capable of resisting $6\frac{1}{2}$ tons safely per square inch, 16 rivet *areas* are necessary to resist the 63 tons shear at the ends of 3, 7, 6 or 4; and as each rivet is in double shear, 8 *rivets* are necessary. Similarly, 7 rivets are necessary

in the attachments at the ends of members 10 and 11. How these rivets are obtained can be seen in Fig. 88.

The verticals, such as 8, have been made of the smallest paired bars which are practicable, and give an ample margin of safety.

CHAPTER XII.

ROOFS—DEAD LOAD ONLY.

Although in roofs of large span and in exposed situations it is a matter of necessity that provision should be made to resist the pressure of the wind, yet, in the ordinary cases which occur in the practice of every architect, it is more customary to design the members to resist a vertical dead load only, a sufficient allowance being made to cover not only the weight of the material employed, but that of a considerable covering of snow also, with a slight addition to allow of ordinary wind pressures being sustained. It is thus a common thing to calculate for a total vertical dead load of 60 lbs. per square foot, this being sufficient, under normal conditions, to fulfil all these requirements.

Such a load, spread over the whole surface of a roof, is conveyed by the common rafters to the purlins, and by them to particular points on the principal rafters. The common rafters thus have small loads to convey across short spans—from purlin to purlin, and though in large roofs they have to be calculated as beams bearing distributed loads, usually their sizes are determined by "rule of thumb," simply because there are practical difficulties in the way of procuring and securing rafters, whether of iron or wood, which contain only the amount of material theoretically necessary. Purlins, however, should always be calculated, and even in roofs of moderate span they

often have to be trussed to enable them to convey the

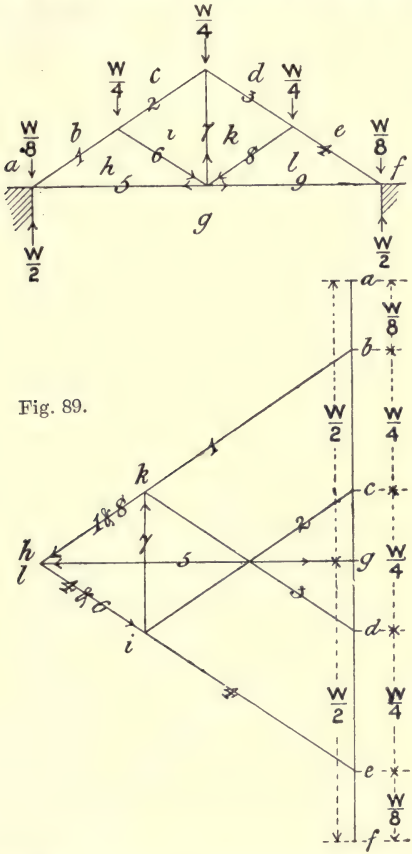


Fig. 89.

loads brought upon them, without their being made of inconvenient depth.

The main trusses of all roofs should be calculated, yet

it is a common practice for architects to depend upon tables of scantlings, to be found in various textbooks, and

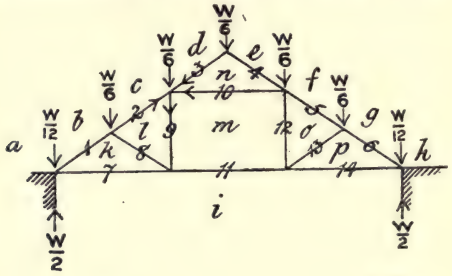
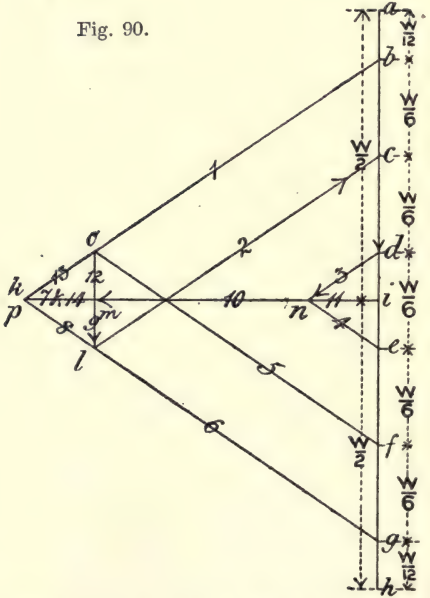


Fig. 90.



upon their own ideas of what ought to suffice. There is some excuse for this where wood is the material employed,

as timbers which would be of sufficient scantling to resist the direct stresses brought upon them, would be insuf-

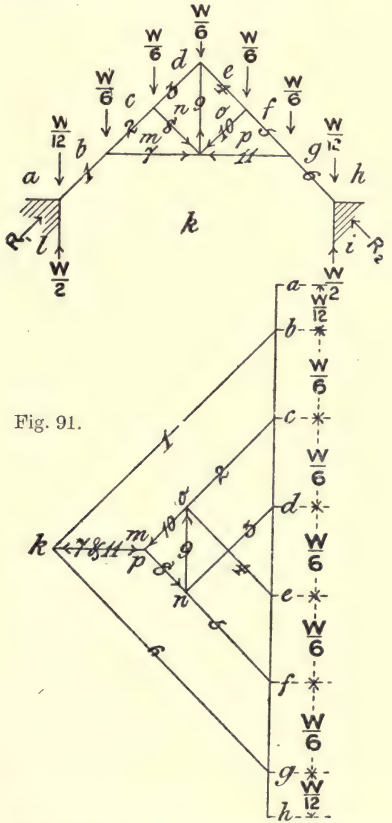
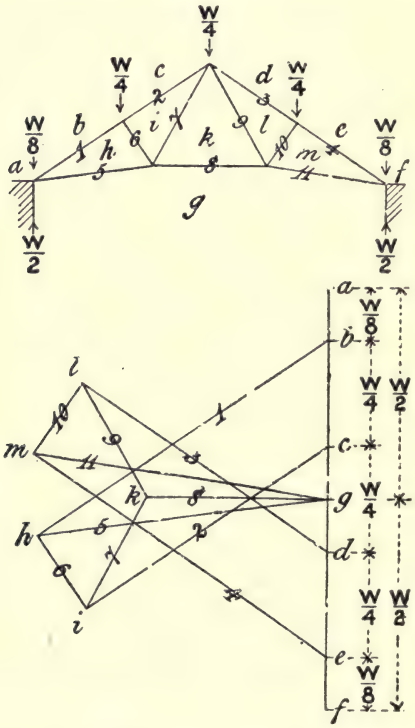


Fig. 91.

ficient for the formation of good joints at their extremities. The amount of timber required at the joint thus fixes the scantlings throughout, for it would be the merest waste

of labour and material, unless necessary for æsthetic effect, to cut away from the remainder of the member till only the theoretically necessary amount of timber were

Fig. 92.



left to resist the direct stress. With iron, a comparatively expensive material, and easily forged to any required form, it is a different matter. The stress upon each member and every joint, according to the circumstances

of each particular case, should be accurately determined, and sufficient metal—no more—should be provided to meet it.

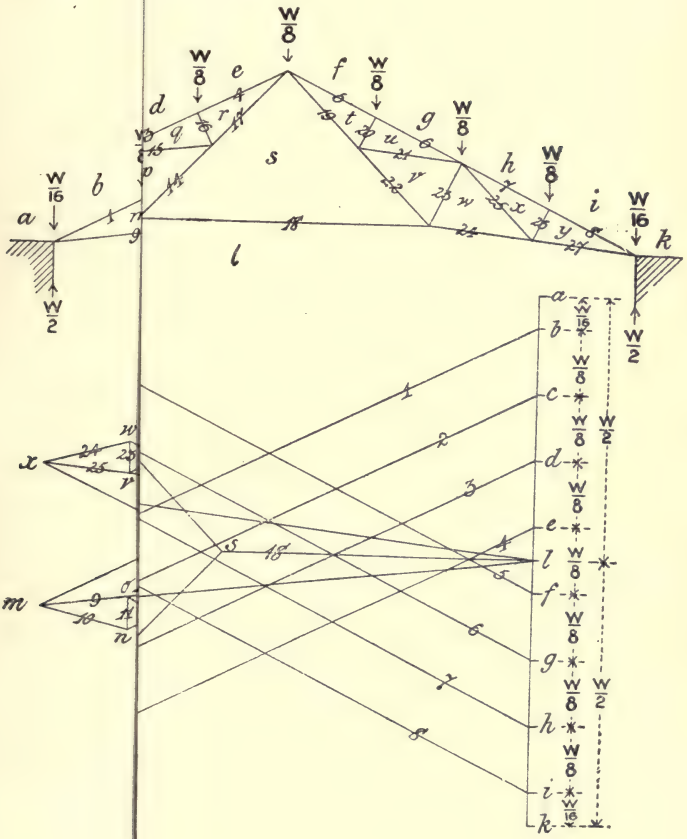
Few cases present any difficulty so far as the determination of the stresses in the members is concerned. Figs. 89 and 90 illustrate the ordinary king- and queen-post roofs. No explanation of the diagram is necessary to those who have read the previous chapter; but it may be pointed out that in the member 11, Fig. 90, the tensional stress $m\,i$, as shown on the diagram, is that which would be brought upon it if no straining piece were used. The introduction of a straining piece would cause the tension in the tie-beam to remain uniformly equal to $k\,i$ throughout, while the straining piece itself would be placed in compression to the extent of $k\,m$. The direction-arrows are only shown round the foot of the king-post in Fig. 89, and the head of the queen-post 9 in Fig. 90. The horizontal distance between the points of application of the purlins is assumed to be uniform in each case, the proportions of the total load acting upon each joint being accordingly uniform also. If it be necessary in any particular case to space the purlins unequally, care must still be taken that the joints shall be vertically under the point of application of the purlins, and the proportions of the loads falling over each joint should be carefully ascertained. If the purlins do not rest directly over the joints, great inequalities of loading are the result, and the principal rafters are immediately put into the position of beams carrying fixed loads which they distribute to the joints.

The case shown in Fig. 91 requires somewhat further consideration. The load lying over the abutment is directly resisted vertically by the abutment; but the thrust on the lower part of the principal rafter, not being

counteracted by a tie-beam, is met by a reaction equal to it in amount and opposite in direction. The readiest method of ascertaining this is to consider the truss as ceasing with the collar-beam, the reactions at the feet being in the directions of the principals. By drawing *g k* parallel to 6 and *b k* parallel to 1 till they meet in *k*, the polygon of external forces *b c d e f g k b* can be completed, giving *g k* as the reaction or thrust along the principal rafter 6, and *b k* as that along 1, to the scale of weights employed. The total thrust on the abutment is equal to the resultant of *b k* or *g k* and $\frac{W}{12}$, which has to be met by the weight of the wall, or, if this be of itself insufficient, by a buttress.

Where this form of roof is employed there is often a curved rib (solid) attached to underside of the collar and principals for the sake of appearance, and it is very commonly supposed that if such a rib be employed it will in some manner act as a tie and obviate the necessity for a buttress. This, however, is an entire mistake. This rib may (or may not, for even this depends upon the construction, and the rafter is usually strong enough without additional help) add to the stiffness of the rafter; but it cannot change the direction or lessen the amount of the thrust against the top of the wall until such thrust becomes active by the spreading out of the walls, and then only if it be very carefully jointed. It will be noticed from the direction-arrows that the collar, which is commonly thought to act as a tie, and to serve to a small degree the purpose of a tie-beam at the foot, is actually in compression. It prevents sagging of the rafters under the weight brought upon them, but does not tie in their feet unless the rafters be so strong as to carry the weight acting at the junction of collar and rafter

Fig. 94.



[*Between pages* 128 *and* 129.

without assistance, in which case an entirely different diagram would be necessary, and the respective loads would have to be considered as acting at the head and feet of the truss and over the struts only.

It is to be noticed that in king-post trusses there is no theoretical necessity for the king-post to be attached to the tie-beam at all, the only advantages gained by making this attachment being the prevention of sagging in the tie-beam from its own weight—until wind force is taken into consideration, when the value of the tie and king-post being joined is immediately felt.

The ordinary form of iron roofs shown in Figs. 92 and 93, with their many varieties, require little consideration, the stress diagrams being readily ascertained; but it is to be noted that the cambering of the tie-rod, which is customary, while it reduces the length of the struts, adds considerably to the stresses upon both the rafters and ties, and is thus by no means an unmixed blessing. The better forms of iron roof are therefore clearly those of the same type as that shown in Fig. 92, which, without adding greatly to the stresses upon the rafters, reduces the length of the struts to a minimum.

This type of truss can be adapted for a large span, as shown in Fig. 94, the determination of the stresses being still fairly simple, in spite of three unknown stresses being met with when the strut 13 is reached, whether the members at its head or its foot be considered. These can be found by making a trial diagram (dotted in Fig. 94) to ascertain the stresses on the members 16 and 15. Whatever may be the amount of stress on the rafters 3 and 4, it is clear that $q\ r$ will denote, both in amount and direction, the stress on the strut 16, and by drawing lines from its extremities q and r parallel to 15 and 14 respectively, a point is found at their junction which will

represent the position of p as compared with q and r. It is then only necessary to slide the triangle $q\ p\ r$ up the lines $q\ d\ r\ e$, maintaining the inclinations of its three sides till the line $o\ p$ is met by the apex p, to determine the length of $o\ p$ (which scales off the stress upon the member 13), and so to reduce the number of unknown forces about either the head or foot of 13 to two, thus enabling the diagram to be completed.

CHAPTER XIII.

ROOFS—WIND PRESSURES AND COMBINED LOADS.

The only live load to which roofs are to any extent exposed under ordinary circumstances is the pressure of the wind; but this is of such importance, and may be considered in so many different ways, as to demand a chapter to itself. The different conditions which may arise, with the varying methods of treatment which they necessitate, are most easily compared by retaining the same type of roof for each illustration, and a very common and useful form of iron roof (see Figs. 95 to 99) has been chosen for the purpose, the same system of investigation as that employed being, however, applicable to all types alike.

The most natural assumption is that the wind acts horizontally against one side of the roof, though this in reality does not often occur. Each side of the roof is exposed in its turn to the wind, and the members have to be designed to meet the greatest stress brought upon them in either case.

In Fig. 95 it is assumed that the wind acts horizontally against the left side of the roof, and that the feet of the rafters are both fixed to the abutments—in which case it is only necessary to reverse the diagram should it be required to ascertain the stresses due to a wind from the right.

The only difficulty met with is that of ascertaining the amounts and directions of the reactions. It is evident

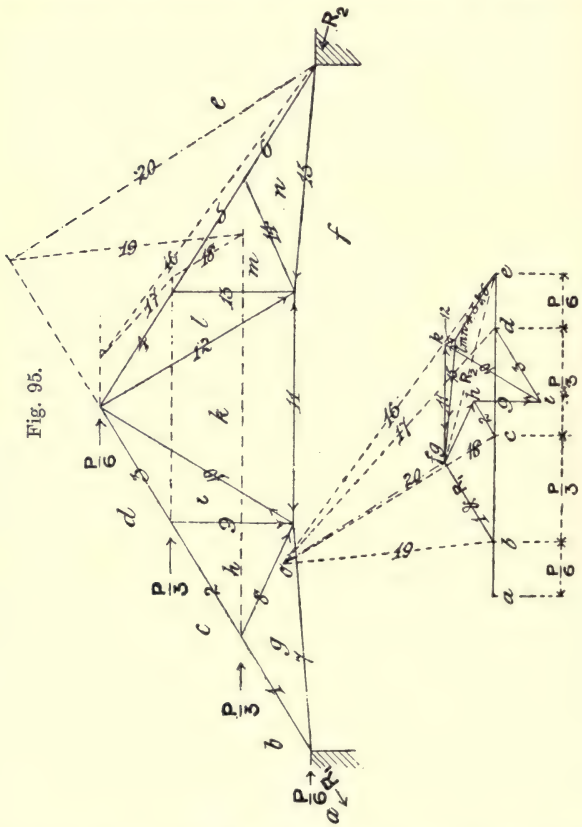


Fig. 95.

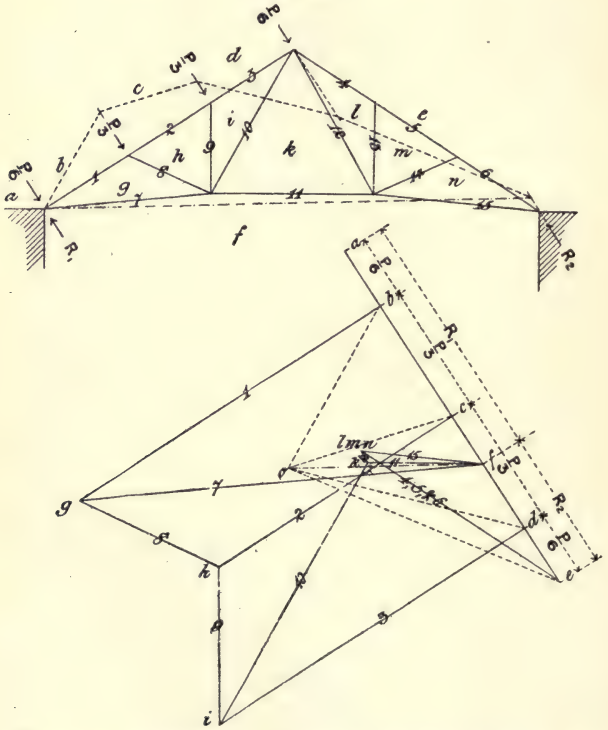
that the pressure $\frac{P}{6}$ which lies between the spaces *a* and *b* is resisted directly by the abutment: it may therefore

be eliminated from the problem, the other pressures only being considered. These other pressures place the members 1 and 2 in *tension*, with the result that the abutment must pull against the rafter to the same extent as that to which it is strained.

This gives a reaction *downwards*, in the same direction as the rafter, a sufficient weight having to be provided attached to the shoe, to supply this reaction. Having thus determined the direction of the reaction of the left-hand abutment, the polar diagram and funicular polygon can be used to ascertain its amount, and also the value, both in direction and amount, of the other reaction. Taking any pole *o*, join *o e*, *o d*, *o c* and *o b*, numbering these lines 16, 17, 18 and 19 respectively. The funicular polygon must be started from the point where the attachment is made between the roof and the right-hand abutment, for there only is it certain that R_2 will be met, its direction not being known. From that point draw a line 16 parallel to 16 on the diagram, till the line of direction of $\frac{P}{6}$, lying between *e* and *d*, be reached. Continue the polygon with 17 and 18 parallel to 17 and 18, till the lines of the pressures lying between *d* and *c* and *c* and *b* be reached respectively. Then draw 19 parallel to 19 till the direction of R_1 be reached, in the same line as the members 1, 2 and 3. Close the polygon with line 20, and from *o* on the diagram draw 20 in a parallel direction till a line from *b*, parallel to the known direction of R_1, be reached in *f*. Then (the effect of $\frac{P}{6}$ lying between *a* and *b* being neglected) *b f* will scale off the value of R_1, and *f e* will denote R_2, both in amount and direction. Once the point *f* is determined, the remainder is simple enough, upon the usual lines. The

nature of the stresses, however, as shown by the arrows round the joints, is instructive. The members 1, 2 and 10 are in tension, 7, 13 and 14 are unstrained, and all the

Fig. 96.



rest are in compression. This shows that if 11 and 15 be designed as tension rods, they would not suffice to meet the compression brought upon them by a strong horizontal wind. The easiest practical way to meet this

difficulty is to add to the weight of the roof covering in exposed situations, lightly-constructed roofs in such positions being a mistake. The compression in the rafters 4, 5 and 6 is thus increased somewhat, but 11 and 15 can still be retained as ties and used in considerable length.

Except under rare conditions, however, the wind does not act horizontally, but beats either upwards or downwards. When a building is properly closed in, an upward-acting wind, even of considerable power, has but little effect upon the roof, though in open sheds the case is different, and the possibility of an upward eddy has to be taken into account. Downward pressures are common, and exercise their greatest force when acting normally to the slope of the roof. The following is the generally accepted allowance per foot super of surface exposed for different inclinations :—

Pitch of Roof.	Normal Pressure of Wind in pounds per foot super.	Pitch of Roof.	Normal Pressure of Wind in pounds per foot super.
5°	5·0	50°	38·1
10°	9·7	60°	40·0
20°	18·1	70°	40·0
30°	26·4	80°	40·0
40°	33·3	90°	40·0

In Fig. 96 is shown a roof exposed to such a normal pressure upon one of its sides. If the heels of both rafters be fixed to the abutments, as is generally the case in small roofs, and is presumed in Fig. 96, the reactions will both be opposite in direction to the wind pressure. Their values are found most easily by means of the polar and

funicular polygons, which are shown in dotted lines, the proportion of the pressure, $\frac{P}{6}$, which acts directly on the left-hand abutment, being disregarded, as evidently no portion of it is conveyed to the right-hand support. The polygon might in this case have been started from *any* point in the direction of R_1, the reactions and the pressures being parallel, and the same closing line, or one parallel to it, would have been obtained, fixing the same point *f* on the diagram to represent the space *f* on the sketch. The reaction R_2 lying between the spaces *e* and *f* is now shown, both in value and direction, by the line *e f* on the diagram, and R_1 by the line *f a*. The point *f* once ascertained the rest is easy, the stresses upon the various members being found in the usual way.

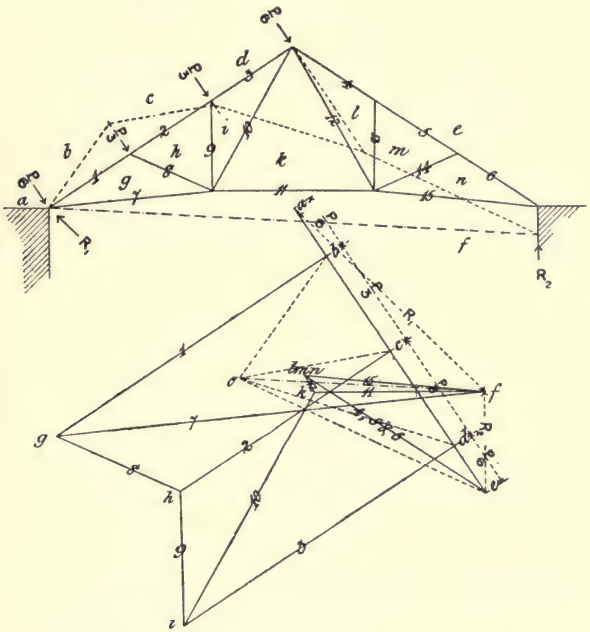
It would have been possible to have ascertained the value of the reactions upon the principle of moments arithmetically; but this is now rarely done save in important structures as a check upon the graphic calculations, which, however, prove themselves by the closing of the diagrams if carefully drawn.

Figs. 97 and 98 are illustrative of cases in which one of the principals only is fixed to its abutment, the other resting upon rollers so as to allow of expansion. In such cases the inclined reaction is all taken up by the fixed end, there being vertical reaction *only* through the rollers. This is a common arrangement in large roofs when the expansion and contraction of the members (if constructed of iron), which is due to variations of temperature, has to be provided against.

In Fig 97 it is presumed that the principal is fixed to the left-hand abutment, the rollers being on the right. The reaction at the right abutment (R_2), therefore, is vertical; but all that is known of R_1 is that it passes

through the point of attachment of the principal and the left abutment. The funicular polygon may therefore be started from that point with safety, and continued to a vertical line representing the reaction whose direction is known. The closing line, when transferred to the polar

Fig. 97.

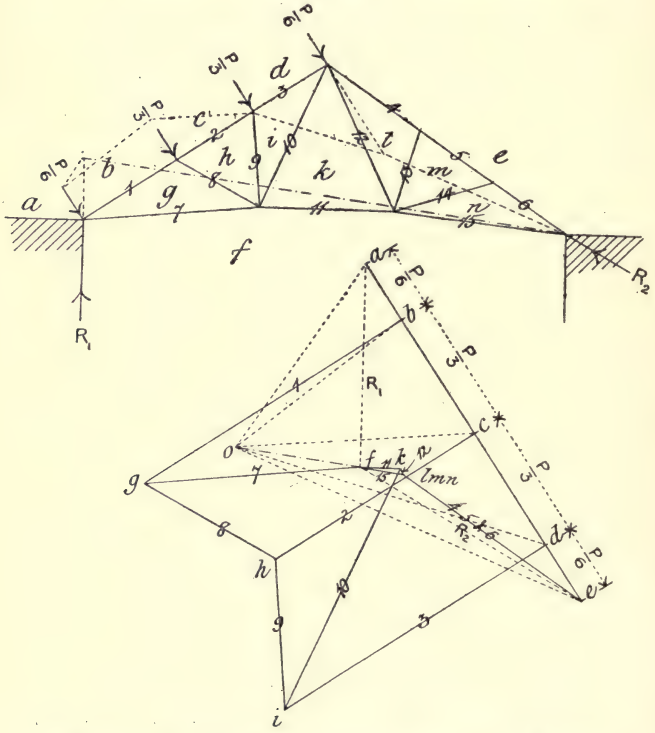


diagram, is also drawn to meet a vertical line through e at f, giving $e\,f$ as the vertical reaction R_2, the other reaction, R_1, being equal both in direction and amount to $f\,a$.

The method of ascertaining the reactions in Fig. 98 is precisely similar, save that the funicular polygon is

started from the point of attachment of the principal and the right-hand abutment, and is drawn till a vertical line through the left-hand abutment is reached giving R_1 in

Fig. 98.



the diagram as a vertical force from *f* to *a*, and R_2 as an inclined reaction from *e* to *f*.

In both cases, once the point *f* is found on the diagram, and the reactions determined, the rest is plain sailing,

and the stresses upon the members, both in direction and value, can be ascertained as usual, the closing of the diagram proving its correctness.

Fig. 99 shows the combination in one diagram of the stresses produced by a vertical load and a horizontal wind pressure acting concurrently.

The first thing necessary is to determine the actual forces acting at the various points. Between a and b there acts a horizontal pressure $\frac{P}{6}$ and a vertical load $\frac{W}{12}$. These being combined give a resultant of F_1. From a to b on the diagram, therefore, set down a line equal to scale, and parallel in direction to F_1. Similarly, F_2 is the resultant of the pressures acting between the spaces b and c, F_3 the resultant of those acting between c and d, and F_4 the resultant of those acting between d and e. From b to c on the diagram, therefore, a line is drawn equal and parallel to F_2, from c to d another equal and parallel to F_3, and from d to e yet another equal and parallel to F_4. The lines from e to f, f to g, and g to h are vertical, being equal and parallel to the loads acting vertically downwards between the spaces e and f, f and g, and g and h respectively.

If the principal be fixed to both abutments, it is evident that F_1, lying between a and b, and $\frac{W}{12}$ lying between g and h, are met directly by the resistance of the abutments on which they respectively act; and that they may therefore be neglected in considering the reactions which affect the structure. By joining $g\ b$ on the diagram, the polygon $b\ c\ d\ e\ f\ g\ b$ is closed, this representing the external forces acting upon the truss, the line $g\ b$ giving the value of the sum of the reactions both in amount and direction. The direction of the reactions being thus ascertained, a funicular

polygon with its reciprocal polar diagram can readily be set up (as shown by dotted lines) to determine how

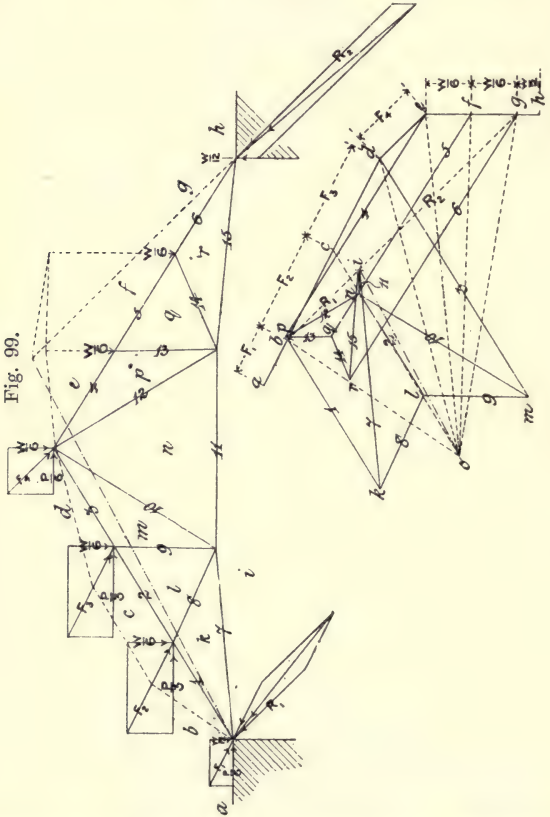


Fig. 99.

much of the total reaction $g\ b$ is operative at each abutment, $g\ i$ being found to be the value of R_2, and $i\ b$ that of R_1. These reactions are irrespective of those caused by

the forces acting directly upon the abutments, the composition with these being shown on the sketch, the resultants in the parallelograms there given representing the forces which the abutments would have to be designed to meet.

When the point *i* has been determined, the diagram of stresses can be formed in the usual manner, and it will prove its own correctness by closing at the finish. It will be noticed that, in the example taken, sufficient dead load has been assumed to so far counteract the wind pressure as to place the members 11 and 12 in tension. The member 12 would, however, be designed to resist the tension found to exist in 10, in case of the wind acting in the contrary direction to that calculated for; and, the same system being pursued throughout, the roof would be symmetrical in construction, each member being made capable of resisting the greatest stress that would be likely to be brought to bear upon it under any ordinary conditions.

CHAPTER XIV.

METHOD OF DESIGNING A STEEL SEGMENTAL ROOF.

In Figs. 100 and 101 all the necessary work is shown in order to ascertain the stresses upon a segmental roof of the "Warren" pattern, over a theoretic span of 60 feet, when subject to a depressed wind pressure of 40 lb. per vertical foot super (or rather to the normal equivalent of 26·4 lb. per foot super of surface exposed for an inclination of 30°), and to a dead load of 20 lb. per horizontal foot super. These pressures are compounded at the various joints, as explained above; but the resultants only are shown for the sake of clearness, both direction and value in lb. being given. For the same reason the lettering and arrows, shown in previous diagrams, are here omitted. The trusses, which are 10 feet apart, are supposed to be fixed at the left-hand abutment, and to rest upon rollers over that at the right, Fig. 100 giving the diagram when the wind is from the left, and Fig. 101 that obtained when the wind is from the right. The following table contains the value and denomination of the stresses upon the various members as scaled off from the diagrams, and from this table the members would be designed.

[*Between pages* 142 *and* 143.

TABLE OF STRESSES upon the members of the segmental roof shown in Figs. 100 and 101 for the span of 60 feet (rise in centre 17′ 4″ and camber of tie-rod 5′ 5″), when subject to a dead load of 20 lb. per horizontal foot super and normal wind pressures of 26·4 lb. per foot super, the trusses being 10 feet apart.

Number indicating Member.	Stress in lb. (wind from left).	Stress in lb. (wind from right).
1	− 13,600	− 6,475
2	− 12,550	− 6,025
3	− 11,900	− 6,550
4	− 11,350	− 7,275
5	− 9,525	− 9,025
6	− 8,900	− 9,475
7	− 8,275	− 10,050
8	− 8,725	− 11,125
9	+ 12,900	− 750
10	+ 700	+ 1,200
11	+ 275	− 1,200
12	+ 13,075	+ 475
13	− 100	+ 1,800
14	+ 1,675	− 1,700
15	+ 12,275	+ 1,925
16	− 1,350	+ 2,400
17	+ 3,075	− 2,400
18	+ 10,400	+ 3,625
19	− 2,050	+ 2,700
20	+ 2,900	− 2,000
21	+ 8,750	+ 5,300
22	− 1,600	+ 1,600
23	+ 2,425	− 625
24	+ 7,150	+ 6,300
25	− 1,325	+ 250
26	+ 1,825	− 225
27	+ 5,500	+ 6,325

NOTE.—The sign + signifies "Tension," and the sign − signifies "Compression."

Examination of the table will show that the maximum tensional stresses in members 9, 12 and 15 are so nearly alike that the highest may be calculated for, and the others made similar to it; the corresponding members 27, 24 and 21 being also made similar for the sake of uniformity and to guard against the risk of the truss being built up the wrong way, though the maximum stresses in these are considerably less. Using steel which may be calculated to resist $6\frac{1}{2}$ tons per square inch safely, either in tension or compression, it is found that a rod $1\frac{1}{8}$ inch diameter is required to meet the stress of 13,075 lb. upon member 12. This is ample to resist the small maximum compressional stress of 750 lb. upon member 9, even when full allowance is made for the tendency of a long rod to buckle under compression, and therefore $1\frac{1}{8}$ inch diameter rods may be used for members 9, 12, 15, 21, 24 and 27.

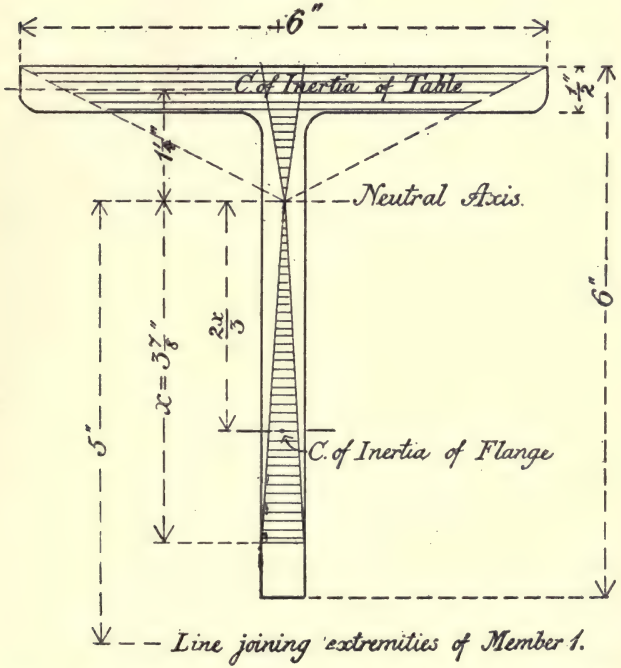
The maximum item (tensional) in member 18 is 10,430 lb., which can be readily met by a rod of 1 inch diameter.

The members 11, 14, 17, 19, 22 and 25, which under dead load only would be in tension, might each be in compression under different circumstances of wind, the maximum being reached in a compression of 2,400 lb. upon member 17 when the wind blows from the right. This member is 13 feet 3 inches long, so that the limits within which the formula given in Chapter VIII. would apply are exceeded, and the results of experiments upon such long compression rods have varied to so great an extent that reliable formulæ have never been devised; but in this instance the stress is very slight, and if $1\frac{1}{8}$-inch diameter rods be used, in conformity with those of members 9, 12, 15, 21, 24 and 27, there will be an ample margin of safety. The maximum tensional stress

is also found in member 17, amounting to 3,075 lb. when the wind is from the left, and this is also very fully met if the rod be $1\frac{1}{8}$-inch diameter.

The members 10, 13, 16, 20, 23 and 26, which would

Fig. 102.



be in compression under a dead load, are sometimes in tension when a wind is blowing, but the compressional stress is the most difficult to resist, and is at its maximum of 2,000 lb. in member 20. Again, $1\frac{1}{8}$-inch diameter rods would amply suffice.

The rafters 1, 2, 3, 4, 5, 6, 7, and 8 present a new problem, for they are curved. The stresses have been tabulated and the diagrams drawn as if they were straight from point to point, and show a maximum compression of 13,600 lbs. on member 1, which, for convenience of calculation, we may call 6 tons without appreciable error. The deflection from the curve amounts to 5 inches in a length of 9 feet. In construction the neutral axis of the member 1 must pass through the pin at each of its ends (or other theoretic point of junction with other members), and also pass through a point 5 inches from the direct line in the middle of its length. This gives a bending moment of 6 tons $\times$ 5 in. = 30 in. tons. Half of this is to be met by the moment of the tension in the table of the T iron of which the rafter would be formed, as taken round its neutral axis, and the other half by the moment of the compression in the flange of the T iron, also taken round the neutral axis.

Assuming the neutral axis of the T iron to pass through a point $1\frac{1}{4}$ inches away from the centre of gravity of its table, as shown in Fig. 102, and T to represent the tension in the table, then—

$$T \times 1\tfrac{1}{4} = 15 \text{ inch tons.}$$
$$\therefore T = 12 \text{ tons.}$$

Allowing $6\frac{1}{2}$ tons per square inch, this calls for a table containing $1\frac{7}{8}$ square inches within the shaded portion above the neutral axis in Fig. 102. This, after allowing for rivet or bolt holes necessary for the attachment of purlins, would be provided by a table 6 in. $\times$ $\frac{1}{2}$ in.

The compressional stress in the flange of the T iron may be supposed to be located at the centre of gravity of the shaded triangle below the neutral axis in Fig. 102. The distance of this point from the neutral axis will be

$\frac{2x}{3}$ if x be taken as the depth of the flange below the neutral axis. Its thickness will be $\frac{1}{2}$ inch, the same as the thickness of the table, as it is usual for T irons to be rolled with table and flange of uniform thickness. The area of the shaded triangle will consequently be $\frac{x}{4}$ square inch, and this may be considered as resisting $6\frac{1}{2}$ tons per square inch safely.

Consequently,

$$6\tfrac{1}{2} \times \frac{x}{4} \times \frac{2x}{3} = 15 \text{ inch tons.}$$

$$\therefore \frac{6\tfrac{1}{2}x^2}{6} = 15$$

$$\therefore 6\tfrac{1}{2}x^2 = 90$$

$$x^2 = 13\tfrac{11}{13}$$

$$\therefore x = 3\tfrac{7}{8} \text{ ins.}$$

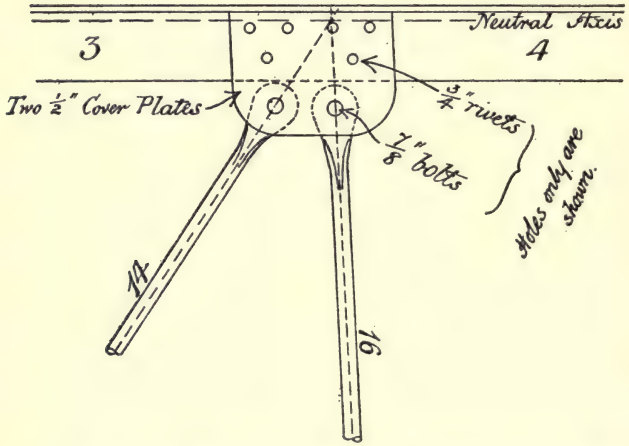
As T irons 6 in. + 6 in. × $\frac{1}{2}$ in. are commonly rolled, these would be used in place of special smaller ones which exactly meet the theoretical requirements.

The reason for considering only the shaded portions of the section given in Fig. 102 as being under stress is that only the extreme fibres will be stressed, when under bending, to the extent allowed for ($6\frac{1}{2}$ tons per square inch), while those about the neutral axis are not under stress at all. The graduation of intensity of stress is quite regular within these limits, and the same result is arrived at by considering shaded portions (obtained as shown in Fig. 102 by dotted lines drawn to the neutral axis centre) to be under full stress, as by considering, as is the fact, that the different fibres are subject to gradually differing stresses.

The rafter would be too long to be continuous along members 1, 2, 3, 4, 5, 6, 7, and 8 without a break, and had

best be cut at the junction between members 3, 4, 16, and 14, and at the corresponding junction between members 5, 6, 22, and 20, there being no bending moment due to curvature at any of the junctions, and the necessary cover plate being ready to hand in the form of the junction plates. A detail of the joint between 3, 4, 16, and 14 is given in Fig. 103. Paired $\frac{1}{2}$-inch cover plates are used, clasping the flange of the

Fig. 103.



T iron rafters 3 and 4, to each of which they are riveted with three $\frac{3}{4}$-inch diameter rivets—more than is necessary to resist the stress in either 3 or 4 as a shear by a large margin, and adopted mainly to secure rigidity such as could not be obtained with single pins.

Pin or bolt connections, however, are formed at the heads of 14 and 16. These members, it will be remembered, are formed of $1\frac{1}{8}$-inch diameter rods, and the ends of these are flattened so as to be clasped by the cover

plates, care being taken that the full sectional area of metal is retained everywhere, and that at the back of the bolt there is enough metal to resist "bearing," or, in other words, that the bolt may not tear out through the flattened end of the rod. To secure this, the longitudinal distance at back of bolt usually equals $1\frac{1}{2}$ times its diameter.

As the bolts are in double shear, each doing double work, $\frac{7}{8}$-inch bolts will suffice. The sectional area of a $\frac{7}{8}$-inch diameter bolt is ·6 inch, and each consequently presents 1·2 square inch to shearing, which is more than sufficient to meet the tensional stress on a $1\frac{1}{8}$-inch diameter rod (containing barely 1 square inch), even at the full allowance; while, in fact, the maximum stress on either 14 or 16 is 2,400 lb., which is met many times over.

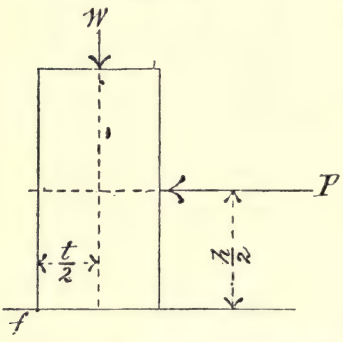
Similar arguments control the designing of all other joints, double plate connections being in all cases the best, with the centre lines of the various members arranged so as to meet at single theoretical points (as shown by dotted lines in Fig. 103), and with simple shears upon the connecting pins equal to the direct tensional or compressional stresses in the connected members.

CHAPTER XV.

WALLS AND CHIMNEYS TO RESIST WIND PRESSURE ONLY.

VARIABLE as is the force of the wind and the direction of its pressure, acting sometimes horizontally and sometimes in eddies, upwards or downwards, it has been found to be sufficiently accurate for most practical purposes, when dealing with the thrusts on walls and chimneys, to assume it to be a horizontally-acting uniform force. Its value can only be found by experiment in the particular situation where the erection under consideration is to be placed; but as a general rule, and in most situations, a pressure of 40 lb. per foot super of the surface exposed to its action may be assumed with safety as the maximum ever likely to occur, though in exposed situations a higher value of 50, 56, or even 60 lb. per foot super is often calculated for.

Fig. 104.

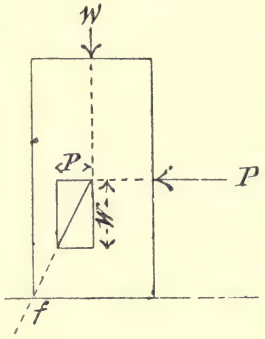


This pressure may, too, without sensible error, be assumed to act at the centre of gravity of the surface pressed against, and in a horizontal direction.

In the case of an ordinary rectangular wall of masonry or brickwork, this centre of pressure is found to lie at a point half-way up the height of the wall.

Suppose, for example, that the wall shown in Fig. 104 is hinged at the point f, and so capable of overturning at the joint at its base, the resistance of the mortar not being taken into account, then the moment of pressure tending to overturn the wall is $P \times \frac{h}{2}$, the value of P being the pressure of the wind per foot super $\times$ the surface exposed in square feet.

Fig. 105.



The moment of stability, tending to cause rotation in the opposite direction, and so to preserve equilibrium, is equally obviously $W \times \frac{t}{2}$, the value of W being the weight of the wall.

Therefore, when there is just equilibrium and no more—

$$W \times \frac{t}{2} = P \times \frac{h}{2}.$$

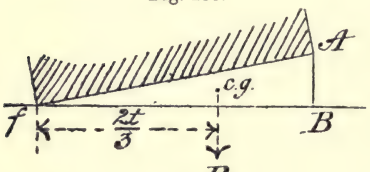
Of course, under normal circumstances the moment of the weight will exceed the moment of pressure (P being taken at maximum), and then any excess of tendency to rotate caused by W is met and equated by the reaction of the earth.

It is obvious also, that when the above equation exactly holds, if the two forces W and P be set out, as shown in Fig. 105, to scale in the directions in which they act, from

the point where their directions cross, and a parallelogram of forces be found, their resultant will, if continued, exactly pass through the point *f*.

Another consideration now forces itself upon us. As soon as the moment of pressure exceeds the moment of weight, and rotation commences around *f*, the resistance to tension of the mortar in the joint comes into play. This it does, as shown in Fig. 106, being most strained where the joint opens most, at A B, while it is *nil* at *f*.

Fig. 106.



At A B the tension must not exceed what the mortar is calculated to bear safely, so that the whole surface upon which the wall rests may be considered as having a layer of mortar having *half* this power of resistance spread over its surface, and this resisting force to tension may be considered to be collected at its centre of pressure, which is the centre of gravity of the triangle A *f* B, and to act vertically, helping W to counteract the overturning power of P.

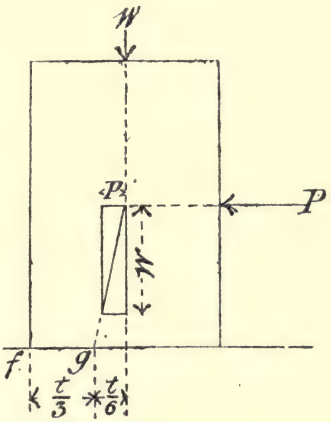
The moment of resistance of the mortar (R) thus becomes $R \times \frac{2t}{3}$ (see Fig. 106), R passing vertically through the c.g. of the triangle A *f* B, and the total equation when there is just equilibrium is now—

$$\left(W \times \frac{t}{2}\right) + \left(R \times \frac{2t}{3}\right) = P \times \frac{h}{2}.$$

When considering a wall of any length it is usual to calculate for one foot run of it only, and then either W,

R, P, t or h can be found if all the others are known (and, as a rule, in practice it is t which is required, h being fixed by circumstances), for W is dependent upon t and h, R upon t, and P upon h. W, for example, is equal to the weight per foot cube of the material to be used $\times t \times h \times$ 1 ft. run of wall (t and h being taken in feet), and this can be substituted for W in the equation if necessary. Similarly P is equal to the wind pressure per square foot $\times h \times$ 1 ft. of wall (h being taken in feet), and R = the safe resistance *per square inch* of the mortar to be used $\times t \times$ 12 in. run of wall (t being in this case taken *in inches*).

Fig. 107.



The *safe* resistance of ordinary lime mortar to tension cannot be taken at more than 5 lbs. per square inch, though cement mortar, 1 to 1, may be calculated at 20 lbs.

It is, however, usually considered that the mortar should never be allowed to come into tension, and to avoid this, and produce an uniformly varying intensity of pressure upon the joint, the resultant of W and P must pass through a point g, situated at a distance usually taken as $\frac{t}{3}$ from f, or of $\frac{t}{6}$ from the middle of the joint, in long walls or solid blocks of masonry which are rectangular or square on plan (see Fig. 107). The

moments then have to be taken about this point g, and the equation of equilibrium becomes—

$$W \times \frac{t}{6} = P \times \frac{h}{2}.$$

For blocks of masonry of other plans, the following are the positions at which the point g is taken—

Solid circles: $\frac{3t}{8}$ from f, $\frac{t}{8}$ from middle of joint.

Thin hollow squares: $\frac{t}{6}$ from f, $\frac{t}{3}$ from middle of joint.

Thin hollow circles: $\frac{t}{4}$ from f, $\frac{t}{4}$ from middle of joint.

These two last values apply to square and round factory chimneys.

Of course, when dealing with such structures as tall chimneys, in which the ring of masonry diminishes in thickness from the bottom upwards, the base course of each thickness has to be taken as a point of calculation, and as many separate calculations made as there are differences in thickness.

In a structure which is not rectangular on plan, too, the wind is found *not* to act as if its full force were exerted against a vertical plane through the axis of the structure, and at right angles to its direction. It acts thus against surfaces which directly oppose it; but the canted sides (which are generally similar on either side of the axis) offer less resistance to its action, and it may in practice be considered that the wind possesses only half its normal intensity against such sides, supposing them, as just said, to be symmetrically placed.

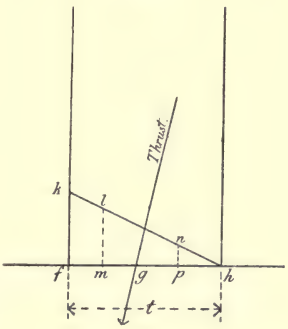
Carrying this to its logical conclusion, in the case of a structure which is circular on plan, the pressure of

the wind against it is only half what it would be against a vertical plane drawn through the axis of the structure.

It may be instructive to notice, however, that although the distance fg is, as shown in Fig. 107, usually taken as $\frac{t}{3}$ for practical convenience, this is not quite correct theoretically.

Suppose (Fig. 108) that g be correctly located. Then the pressure on fg will equal the pressure on gh. The pressure at h will be nil, and the pressure at f may be represented as fk, when the pressure at intermediate points along fh will be represented by ordinates erected from fh to kh.

Fig. 108.



Then if fg be bisected at m, and gh at p, ml will represent the average intensity of pressure on fg, and pn the average intensity on gh.

The total pressure on fg will be $ml \times fg$, and that upon gh will be $pn \times gh$, and these will equate. That is:—

$$ml \times fg = gh \times pn,$$
$$\therefore ml : pn :: gh : fg.$$

But, by similar triangles,

$$ml : pn :: mh : ph.$$

Therefore $gh : fg :: mh : ph,$

or $gh \times ph = mh \times fg.$

This equation is only satisfied if $fg = \frac{t}{3{\cdot}41}$. Consequently, although it is incorrect to assume it, for convenience of calculation, to be $\frac{t}{3}$, any error is on the safe side, as theoretically the point g occurs nearer to the outer edge than it is customary to take it.

CHAPTER XVI.

FOUNDATIONS.

JUST as it is obvious that the stress upon no part of a structure should be greater than can be safely met by the substance of which it is composed, so it is equally obvious that the eventual pressure carried down to the foundations should be spread over a sufficient area to enable it to be safely borne by the soil beneath. In the case of ordinary light structures these conditions are reasonably met by the building regulations in force in most parts of the country, to the effect that the bottom course of the footings of a wall shall be twice the minimum thickness permitted for the wall itself, this extra thickness being gradually obtained by regular offsets, the height from the bottom of the footings to the base of the wall being at least equal to two-thirds of the thickness of the wall at its base. There are no regulations about foundations, but it is usual to provide a solid block of concrete below the footings whose width is at least three times the thickness of the wall, and whose depth is equal to two-thirds of thickness of the wall.

In calculable cases, where heavy pressures have to be met, it is not sufficient to depend upon a rule of this sort.

Whenever practicable—and it generally is practicable—the base of the concrete block should be at right angles to the calculated resultant pressure; and this resultant should be made to pass, not merely within the centre

third of this base, but actually through its centre, by spreading the concrete more on one side of it than the other if necessary, in order to secure this result. The effect of this is that the earth is made to offer a direct resistance to the resultant pressure, evenly distributed over the bearing surface. The necessary extent of this surface is then easily calculated from the following table of the safe bearing capacity of different soils :—

Nature of Soil	Safe Bearing Capacity, per square foot, in tons.
Soft soils, such as bog, marsh, silt, peat, muddy clay, and alluvial deposits in river beds ...	$\frac{1}{4}$
Rocky alluvial river beds, firmly deposited, and diluvial clay river beds	$\frac{1}{2}$
Loams and loamy soil, clay loams and damp clay	$\frac{3}{4}$
Soft impure chalk	1
Loose sand in shifting river bed	$2\frac{1}{2}$
Hard white chalk	$2\frac{1}{2}$
Ordinary superficial sand beds	$2\frac{1}{2}$
Upheaved beds of different sound clays	3
Firm sand in river bed secure from scour, and at depths below 25 feet	$3\frac{1}{2}$
A mixture of solid clay and fine sand and sound yellow clay, containing little water ...	4
Firm sand in estuaries	$4\frac{1}{2}$
Solid blue clay, marl, and firm boulder gravel and sand	5

Clay must in all cases be evenly drained, as whenever and wherever it contains an undue proportion of water its bearing capacity is sensibly reduced.

It is a common practice to assume a safe bearing load of 2 tons per square foot on all ordinarily good soils.

The correct thickness of the concrete bed cannot properly be calculated, as it depends upon several factors which in any particular case must remain indeterminable,

such as inequalities in the resistance of the soil and in the adhesiveness of the materials of which the concrete is made. It is consequently necessary to fall back upon the general rule that, for good Portland cement concrete composed of 1 part of cement to 6 of aggregate, the thickness should equal at least one-fourth the width of the bed.

So far as the footings which rest on the concrete are concerned, little more can be said than that the more fully their bottom course covers the concrete the better, so long as there is room for a man's foot (6 inches will suffice) on either side; and that the offsets should be regular and as gradual as practicable; while necessarily the pressure on no course should be greater than that which the material of which it is composed will safely bear, and its centre should be approximately at the position where it is penetrated by the resultant applied pressure.

The safe load to apply to different substances is ordinarily assumed to be as follows:—

Material.	Safe Load per square foot in tons.
Stock brickwork in ordinary lime mortar	2
,, ,, ,, lias ,, ...	4
,, ,, ,, Portland cement mortar ...	8
Rubble masonry in lias lime mortar	4
Lias lime and ballast concrete, 1 to 6	2½
Portland cement and ballast concrete, 1 to 6 ...	15

CHAPTER XVII.

THE CONSTRUCTION OF A FACTORY CHIMNEY.

THROUGH the kindness of Mr. C. Stanley Peach, in lending a drawing for illustration and in freely supplying information, it is possible to introduce a chapter here dealing with the practical construction of a large chimney.

That selected for illustration, and shown in the Plate (Fig. 109), is a square shaft, 160 feet high, and of 8 feet side internally at top, having a double batter of 1 in 18·35, and built according to the County Council regulations, which require that the thickness of walls shall increase $4\frac{1}{2}$ inches at every 20 feet from the top downwards. This, of course, obviates to a very large extent indeed the necessity for calculating the resistance to wind pressure, but it is very doubtful whether the result is so satisfactory as that which would be secured were theoretical rules properly followed. However, there are the regulations, and at present there is nothing for it but to carry them out.

Strong bricks of the London stock class have been found to be best for the walls, set in lime mortar, and *not* in cement—the latter being too rigid to allow of swaying under a high wind. Tall chimneys and tall steeples always rock, and it has been found by experience to be a mistake to endeavour to stop their doing so.

The lining should be of the very best firebrick procurable, laid in fireclay; but as little of the latter should be

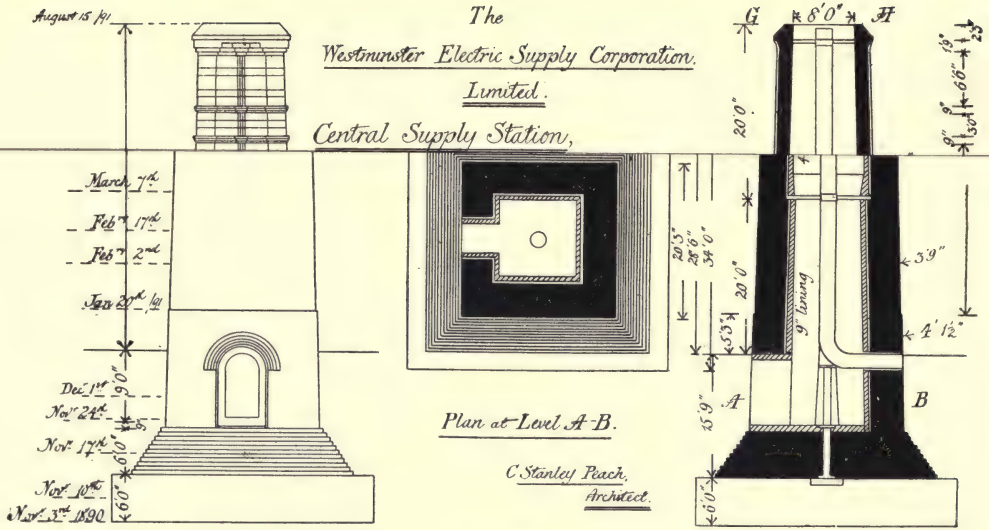


Fig. 109.

[*To face page* 160.

used as possible—only just sufficient, in fact, to cause the bricks to adhere.

Between this lining and the walls there should be an air space of about 2 inches, air being a non-conductor of heat, so that however greatly the lining may become heated, the walls shall be subjected to no excessive temperature. The lining is thus, also, left free to expand and contract without injuring the structure, to which it is *not* bonded, the nearest approach to a bond being formed by introducing headers into the lining, which project into the cavity until they almost touch the outer walls, though they are not allowed quite to do so.

Many designers introduce rows of hoop-iron bond into their walls, but others do not, being afraid of their expanding under high temperature. As already pointed out, however, the walls themselves should never be heated to any dangerous extent, and the objection to the use of bond accordingly disappears.

The lining, it will be noticed, is stopped, and should be stopped, at a little below half the height of the shaft. It is only needed where the temperature is highest; and, moreover, an expansion-chamber is wanted at the half-height, or thereabouts, or else there will not be complete cremation of the fumes. By omitting the lining, therefore, at that spot, the internal dimensions of the shaft are increased, and a sufficient expansion-chamber is provided.

The external form of the chimney, too, is not entirely without its practical effect. By the architect it will be noticed that Mr. Peach has succeeded in giving some architectural character, and this without great additional cost, to even so prosaic an erection as a factory chimney; but a projecting cornice could not be indulged in, from its tendency to interrupt, if not destroy, proper draught.

The steam exhaust, too, is an important consideration: it has to be carried upon a solid stanchion and stayed at intervals; but the stays must *rest* only upon the off-sets of the brickwork, and on no account be built in, as, being of metal, the exhaust-pipe tends to expand quite differently from the brickwork, and if there were rigid connection there would be risk of something going wrong.

As far as the plan of the shaft is concerned, the square form has been found in practice to be the least satisfactory, giving the least draught. The best form seems to be an irregular octagon, or a square with the angles just taken off, possessing almost the capacity of the square, but without its dead angles. A regular octagon is nearly, but not quite, so good.

In spite of much that has been written to the contrary, and of the fact of much extremely rapid chimney building having been done, it is important, unless speed is imperatively demanded, that it should be slowly carried out. Two feet per diem should, for thoroughly satisfactory work, be the utmost allowed, while it is better if the foundations can be allowed to stand for some months, or even a year, before the superstructure is added. In the example given in the Plate it will be seen that the rate of progression was very slow indeed, the heights reached at different dates having been noted as the work proceeded.

CHAPTER XVIII.

RETAINING WALLS SUBJECT TO EARTH PRESSURE.

In dealing with retaining walls, the main difficulty is to ascertain the pressure which is bearing against them, tending to cause them to overturn; and there are two main causes to be considered, those of earth and those of water pressures. By earth pressure is meant the thrust due to normally moist earth, as if it be wet or water-logged through lack of proper drainage at back of, and weep-holes through the wall, the thrust approximates to that from water. This warning is the more necessary, as water pressures, as will be presently shown, are considerably greater than earth pressures, and therefore a wall designed to be amply safe when retaining earth may well overturn when that earth becomes charged with water, for which no means of escape has been provided.

Different classes of earths have very different effects upon a retaining wall, this depending upon the *natural slope* or *angle of repose*, which will be assumed by a heap of this earth when resting upon the ground. The following table, taken from the "R.E. Aide Mémoire," gives the angle of repose of most earths met with in practical work, but where it should happen not to apply, the value can be readily found by a simple experiment.

It will thus be seen that all particles of earth lying within the angle of repose are naturally held in equi-

Class of Earth.	Angle of Repose.	Ratio of Base of Slope to Height.
Fine dry sand	37 to 31°	1·3 to 1, to 1·6 to 1
Sand, wet	26°	2 to 1
„ very wet	32°	1·6 to 1
Vegetable earth, dry	29°	1·8 to 1
„ „ moist	45 to 49°	1 to 1, to ·87 to 1
„ „ very wet ...	17°	3·27 to 1
„ „ punned ...	66 to 74°	·45 to 1, to ·28 to 1
Clay, dry	29°	1·8 to 1
„ damp, well drained ...	45°	1 to 1
„ wet	16°	3·5 to 1
Gravel, clean	48°	·9 to 1
„ with sand	26°	2 to 1
Loose shingle	39°	1·2 to 1
Peat	14 to 45°	4 to 1, to 1 to 1

librium, and are in repose, needing no retaining wall to support them. Particles lying directly beyond this slope, moreover, tend to roll down it, but with a motion and force greatly impeded by friction; while, assuming the back of retaining wall to be vertical, the particles next to the wall tend to fall vertically downwards. Between these two extremes there are all degrees. In any event, however, it may be conceded that the pressure against the back of the retaining wall is that due to the prism of earth above the angle of repose, and that this is resisted partly by the earth below, and partly by the retaining wall.

Of course it is possible for the surface of the retained earth to be either horizontal or inclined at any angle within the angle of repose—or even to be depressed,

though this last case is rare and scarcely ever calculated for. In the case of a horizontal or surcharge surface, however, the following is the method of calculating now generally adopted. Taking the case shown in Fig. 110 as a generally typical one, the wall being rectangular, the earth surcharged to the plane D E, which is within the natural slope, the angle of repose being A B C, it is seen that it is the prism of earth E D B A which is active to cause thrust against the wall. The earth within this prism has, as has been already shown, exceedingly various tendencies to press outwards against the back of the wall D B, and it is a usual and sufficiently correct custom to bisect the angle D B A by the line B F, and to consider the triangular prism of earth D B F as being homogeneous, and to be acting as a wedge against the two plane surfaces D B and B F; it being under these conditions that the greatest thrust is obtained.

The weight of, say, 1 foot run of this prism D B F of the earth under consideration is now ascertained from the following table:—

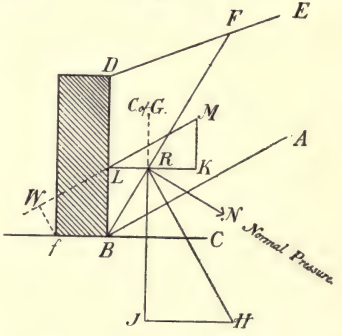
WEIGHT PER CUBIC YARD OF DIFFERENT SOILS.

	cwt.		cwt.
Dry peat	7½	Common gravel	27
Wet peat	15	Wet sand	28
Top soil	20	Gravelly clay	30
Dry sand	22	Rough water gravel	34
Common earth	24	Grey chalk	36
Sandy loam	24	Sandstone	37
Marl	26	Shale	39
Clay	27	Limestone	40

This weight is collected at the centre of gravity of the triangle D B F, and acts vertically downwards until it impinges at R upon the line B F. At this point it is set vertically downwards, as R J, to a scale of weights, and is

resolved out according to the parallelogram of forces into two component forces, one of which, J H, acts horizontally (that is, normally to the back, D B, of the wall). Were it not for the influence of friction, the other should be, as R N, normal to B F; but friction has to be accounted for, and this is done by making N R H equal to the angle of repose, R H becoming, in direction and amount, the corrected component of R J which is taken up by the earth below.

Fig. 110.



The horizontal component J H acts through R, and is consequently transferred to R, and is shown as K L. Where it impinges upon the back of the wall at L, correction for friction has again to be made, by setting the angle K L M equal to the angle of repose. Completing the triangle of forces K L M, the amount of friction is shown to be M K, while the eventual thrust against the wall, both in direction and amount, is represented by M L.

The moment tending to overturn the wall about the point *f* is therefore M L and W *f*, the distance W *f* being measured by dropping a line from *f* perpendicularly to the line M L continued.

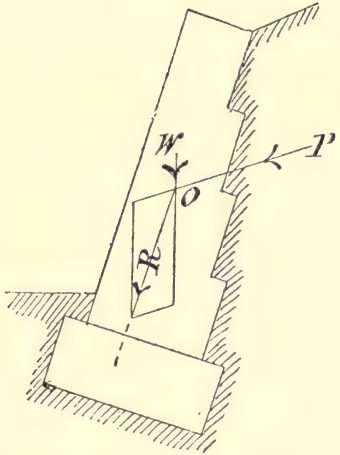
This tendency to rotate of course has to be met by the tendency to rotate in the opposite direction, as measured by the moment of the weight of the wall, also taken about *f*.

For safety, the moments should be taken about *g* (see

Fig. 107), a point located at $\frac{1}{3}$ the base from the outside of wall, instead of at *f*.

So far as the weight of the wall is concerned, this is usually easily ascertained; but it should be borne in mind that if the back be either battered or built in off-sets, all earth lying vertically above the base of the wall acts to assist the wall. Its weight should be added to that

Fig. 111.

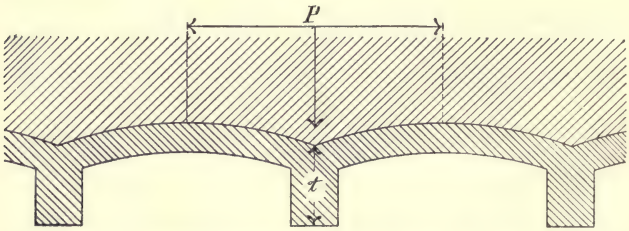


of the wall, and concentrated at the c.g. of wall and of this earth combined.

As to the shape which the section of the wall should assume, this is largely a matter of convenience. The rectangular vertical form is possibly the most common in most work; but its value can be greatly increased by sloping it backwards, so that a vertical line through its c.g. falls further within the base. External buttresses are often used, and, if not too far apart, are valuable additions

as widening the base, overturning having then to take place round their outer edge instead of round that of the wall. Where circumstances do not permit of their introduction, internal counterforts are often used instead; but they can hardly be considered to be so valuable, acting in tension instead of compression. Battered faces, internal or external, are economical in masonry or concrete, but difficult to bond well in brickwork, if the other face be kept vertical. A stepped back with vertical face has much to recommend it; but possibly the best form for brickwork is that shown in Fig. 111. In this the pressure

Fig. 112.



P, as already found, and the weight of wall W meet at O, where a parallelogram of forces is formed, the resultant R of which passes within the centre third of base. The concrete is set at right angles to the direction of R, and the face of wall at right angles to concrete and parallel to R, the back being stepped, but kept parallel to the face.

Retaining walls which are arched on plan between widely projecting buttresses are sometimes employed. The thrusts are concentrated, half the pressure upon each of two adjacent bays (or arches) combining opposite the buttress, which is then designed to meet it, exactly as a wall would be, by equating moments round the outer

edge (for overturning) or round a point distant $\frac{1}{3}$ of the whole thickness within the outer edge (for safety).

The arch is formed of a thin lining of brickwork or masonry only, being designed like ordinary vertical arches, and a certain saving of material eventually results as a rule.

CHAPTER XIX.

RETAINING WALLS SUBJECT TO WATER PRESSURE.

WATER, being a fluid, has an angle of repose of zero, and cannot be surcharged. This is the essential difference between it and earth, while its weight varies so little according to its temperature and ingredients as to be for all practical purposes always 62·5 lbs. per cubic foot. Thus to calculate water pressures is easy to any one who has mastered earth pressures, the same rules being applicable.

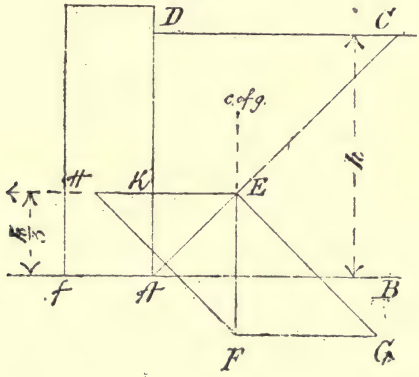
Reference to Fig. 113 will show that A B being the bottom, and D C the top of the water in a reservoir, retained by a vertically-backed wall at D A, and the natural slope being horizontally along the line A B, the complementary angle is the whole angle B A D, and this being bisected by the line A C, according to the rules laid down in the last chapter, the triangular prism of water which is active to cause thrust is that shown in section by the triangle C A D. The weight of this is collected at its centre of gravity, whence it acts vertically downwards, impinging upon the line A C at E, where it is resolved into two forces, E H and E G, of which E H is the force tending to overturn the wall, to the scale to which the weight A F of the triangular prism C A D was set downwards from E.

A little examination of the diagram will show that E F

being vertical, H E horizontal, and E G at right angles to A C, which itself bisects the right angle D A B, therefore the angles E H F and E F H are both angles of 45°, and, therefore, that H E = E F. In other words, the force E H tending to overturn the wall is equal to the weight of the triangular prism of water, C A D.

It will further be seen that the distance A K at which this force acts from the base of wall is exactly one-third

Fig. 113.

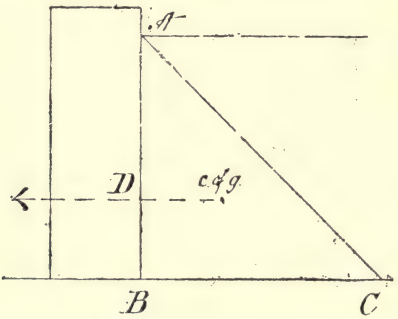


of the depth of the water A D, for the vertical line through the c.g. of the triangle C A D being parallel to A D, and impinging upon A C at E, makes $A E = \frac{A C}{3}$, and similarly, E K being parallel to C D, therefore A K : A D :: A E : A C, which proves the proposition.

It is more convenient, and quite as correct in practice, to reverse the prism, as shown in Fig. 114, and to say didactically that the pressure against the back of the wall A B is equal to the weight of the 45° triangular

prism of water A B C, acting horizontally through its centre of gravity, and thus impinging upon A B and D, D B being equal to $\frac{A B}{3}$. Another way of expressing this same fact is by the statement that the pressure exerted by water upon any surface is equal to the weight of a column of water whose base is equal to the surface pressed, and whose height is equal to the depth of the centre of gravity of the surface pressed below the surface of the water.

Fig. 114.

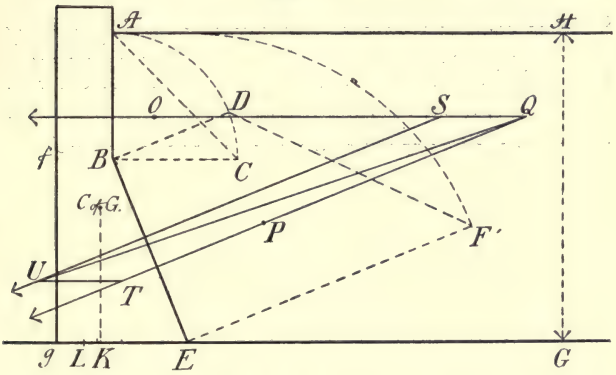


In one very marked respect, however, does water act in a different manner from earth. Being a fluid, it always presses perpendicularly against the surfaces opposed to it. At first sight it looks as if this might be taken advantage of by battering the inside of retaining walls, but this results in bringing the safety point further from the outside of the wall, and in the end it is found best to use retaining walls with a vertical inside face.

It is not, however, altogether unusual to build a wall as shown in Fig. 115, with the upper portion vertical and the lower battered. In such an instance two calculations are necessary—for the upper portion only, as rotating about *f*;

and for the whole wall, upper and lower portions combined, as rotating about g. The upper portion A B may be considered to be pressed against by the weight of the triangular prism of water A B C acting horizontally through its centre of gravity, O. The lower sloped portion B E has a curiously shaped prism of water acting against it, represented by the polygon B E F D, of which B D is drawn at right angles to B E, and is equal to the depth B A of the point B from the surface of the water;

Fig. 115.



while the line E F is also drawn at right angles to B E, and is equal to the depth H G of the point E from the surface of the water. The weight of this prism, B E F D, is collected at its centre of gravity, P, and acts perpendicularly to the inclined surface B E.

To obtain the resultant pressure against the whole wall, it is necessary to combine the pressure of the triangular prism A B C passing horizontally through O, and the pressure of the irregular prism B E F D passing through P, by continuing their directions back till they

meet in Q, and there setting up a parallelogram of forces Q S U T, of which Q S represents the action of the triangular prism in magnitude and direction, and Q T similarly represents the action of the irregular prism, and of which Q U is the resultant.

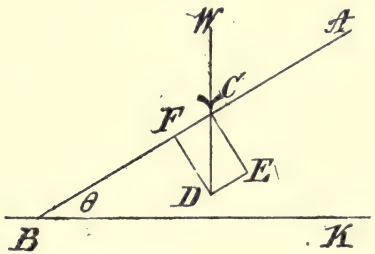
This, of course, has to be combined with the weight of wall acting through its centre of gravity vertically downwards to obtain the eventual resultant, which must pass within the base of the wall as against actual overturning. As already said, however, it should pass well within the base of the wall to secure anything like uniform variance of pressure, but how far within is not always easy to determine. In practice, however, it may be said that if the vertical line through the centre of gravity of the wall cut the base at K, the final resultant should pass between K and L, the line K L being one-third of the line K *g*.

CHAPTER XX.

RESISTANCE TO SLIDING.

FAILURE by sliding, though not very common, is yet possible and of occasional occurrence, and so must be provided against. Of course, were all substances perfectly smooth, and so perfectly frictionless, sliding would inevitably take place, and the more nearly two adjacent

Fig. 116.



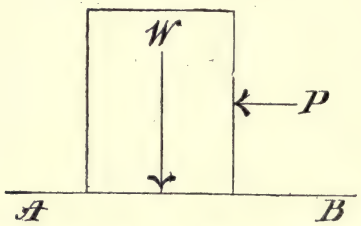
surfaces approximate to perfect smoothness, the greater is the possibility of their sliding upon one another.

Suppose W (Fig. 116) to be a mass of one material resting upon a wedge A B K of another, the wedge being such that sliding is just upon the point of taking place, the angle A B K or θ being the angle of repose between the two substances. The weight of the mass W can be represented to scale by the vertical line C D, and under these

circumstances it is being held in position by the normal resistance of the wedge A B K, as represented by the line C E at right angles to A B, and by the friction, as represented by the line C F, along A B—the lengths of these lines being determined by forming a parallelogram of forces round C D, resolving it into forces C E and C F, normal and parallel respectively to A B.

F C is the value of the friction between the two substances under consideration, which has to be overcome before sliding can take place. Under the present circumstances, it is just met and equated by the component

Fig. 117.



of W which acts along A B, and its value is constant for these two substances, being equal to the normal component C E, multiplied by the tangent of the angle of repose (tan. θ).

This is readily proved, for the angle D C E is equal to the angle θ, and D E = F C.

Therefore
$$\frac{D\,E}{C\,E} = \text{tan. } D\,C\,E$$
$$= \text{tan. } \theta.$$
$$\therefore D\,E = C\,E \text{ tan. } \theta.$$
$$\therefore F\,C = C\,E \text{ tan. } \theta.$$

Tan. θ is known as the *coefficient of friction*, and is a

constant multiplier for any two materials, the angle of repose between which is known.

Thus, if W be the weight of a block of one material resting upon a horizontal surface, A B, of another (Fig. 117), the coefficient of friction between the two materials being known, and the normal component of the weight being in this case equal to the weight itself, then the friction is represented by—

$$W \times \text{coefficient of friction},$$

and the pressure P, parallel with the surface A B, which is necessary to overcome this friction and cause sliding, must consequently be greater than W × coefficient of friction.

The following table gives the generally accepted angles of repose and their tangents (or the coefficients of friction):—

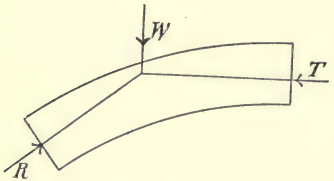
	Angle of Repose.	Coefficient of Friction.
Masonry and brickwork, dry...	31 to 35°	·6 to ·7
Ditto with wet mortar	25½°	·47
Ditto with slightly damp mortar... ...	36½°	·74
Ditto on dry clay	27°	·51
Ditto on moist clay...	18¼°	·33
Wood on stone...	22°	·4
Metals on stone	35 to 16⅔°	·7 to ·3
Wood on wood, dry...	14 to 26½°	·25 to ·5
Metals on metals, dry	8½ to 11½°	·15 to ·2
Smoothest and best greased surfaces ...	1¾ to 2°	·03 to ·036

CHAPTER XXI.

ARCHES AND THEIR ABUTMENTS.

It being the object of this chapter to show how to perform graphically the calculations necessary for the determination of the stability of arch rings and their abutments, it is necessary at the outset to state that the theory relating to arches is in a very uncertain state. There are, in fact, many bridges that do not satisfy the conditions of stability explained in this chapter, but it is found that the best existing examples do satisfy these conditions. The relative dimensions of the various parts of an arch are sometimes given by certain formulæ which have been arrived at by theory, but in practice the graphical method has been found more satisfactory.

Fig. 118.



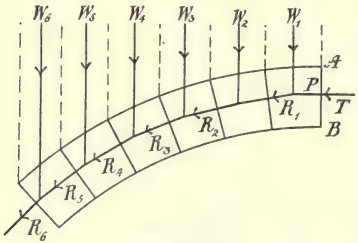


Fig. 119.

It is clear that an arch is kept in equilibrium by the influence of three forces, see Fig. 118, viz., T, the thrust

at the crown; W, the combined weight of the arch ring and its superincumbent load; and R, the reaction of the abutments.

Fig. 119 shows one half of an arch in which W_1, W_2, W_3, &c., represent the combined weights of each voussoir and its load. Assuming the arch to be symmetrically loaded, it is obvious that the thrust at the crown must be horizontal. At what point in the line A B this thrust acts cannot be determined with any degree of accuracy, but it is usual to assume that it acts somewhere within the middle third. Suppose it acts at the point P, then produce its line of action to meet the line of action of W_1 with which it is combined to produce the resultant R_1. Combine R_1 with W_2 to produce R_2. If this operation is continued throughout the whole arch ring a series of lines approximating to a curve are produced. If the arch be imagined to be composed of a large number of thin voussoirs, and the above operation be performed, the series of lines becomes a curve. Such a curve is called a *line of resistance*, or a *line of pressure*. It is clear that if T acts at any other point in A B the line of resistance will occupy a different position to that shown in Fig. 119; in fact, by assuming different points of application of T it is possible to find an infinite number of lines of resistance.

According to theory it is possible for arches to fail in three ways:—

1. By rotation.
2. By crushing.
3. By the sliding of the voussoirs upon one another.

Rotation.—In Fig. 119 it is seen that the resultants R_1, R_2, R_3, pass well within the arch ring, but it is quite possible, if the ring is too thin or the load too heavy, that the resultant may pass outside the arch ring. Thus,

Fig. 120 shows a voussoir with the line of pressure passing through the extrados, in which case it would rotate about the point P; while in Fig. 121 the line of pressure passes through the intrados, in which case the voussoir would rotate about the point Q. The line of pressure, therefore, should always pass well within the arch ring, and in practice it is usual to so construct arches as to permit a line of pressure to be drawn within the *middle third.*

Crushing.—If a line of resistance can be drawn within the middle third of an arch ring it will necessarily be so deep as to present an area in the bed joints of its voussoirs

Fig. 120.

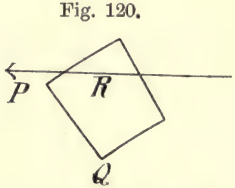
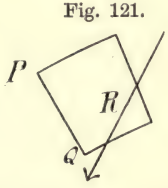


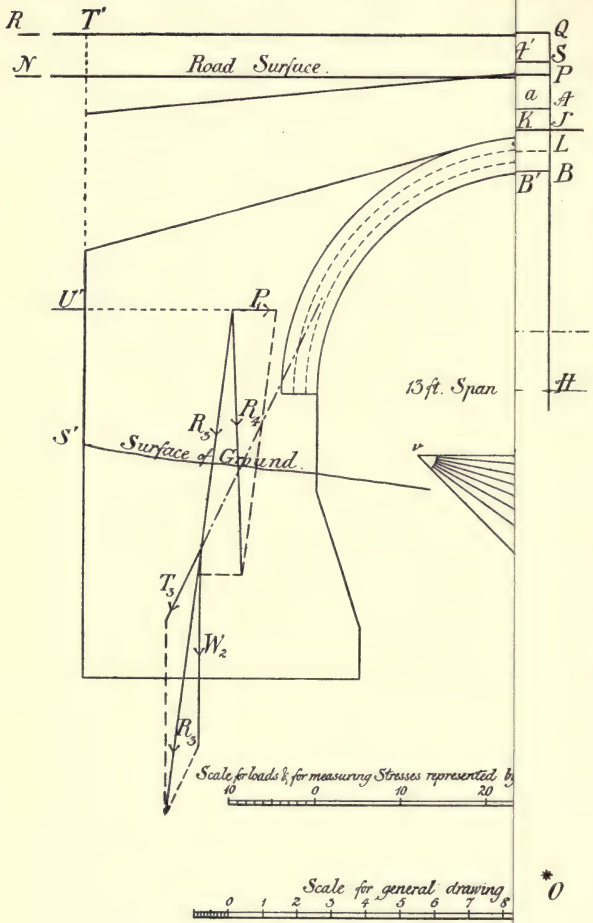
Fig. 121.



many times greater than that theoretically necessary to resist the pressure brought to bear upon them.

Sliding.—An arch is not at all likely to fail by the voussoirs sliding upon one another, because if the line of resistance passes within the middle third it will be almost normal to the bed joint, and more important still is the fact that the blocks are cemented together.

Further than this, the form of an arch exercises considerable influence upon the manner in which it is likely to fail. Thus in a semicircular arch the crown tends to fall in while the haunches tend to rise; in a pointed arch the haunches tend to fall and the crown to rise; while in a flat arch the voussoirs tend to slide upon one another and the joints at the abutments to open.


R
T'
Q
N
Road Surface.
T'
S
P
a
A
K
J
L
B'
B
U'
P
13 ft. Span
H
S'
Surface of Ground.
V
R3
R4
T3
W2
R3
Scale for loads & for measuring Stresses represented by
10 0 10 20
Scale for general drawing
0 1 2 3 4 5 6 7 8
O

If we trace a line of pressure through an arch ring, the points round which rotation tends to take place are those on the intrados or extrados to which this line passes closest.

It has been thought best to demonstrate the method of graphically calculating for the stability of an arch by taking an example. For this purpose a bridge with a low central arch of 30-ft. span, radius 20 ft. 6 ins. and 6 ft. 6in. rise (rise should not be less than $\frac{1}{6}$ span) has been chosen, spanning a stream or cutting, with semi-circular arches, radius 6 ft. 6 ins. on either side.

The shape of the arch, which is governed by such considerations as the nature of the space to be spanned, expense and taste, must in all cases be determined first of all. It is symmetrically loaded, and composed of stone weighing 160 lb. per cubic foot, with an earth filling of 100 lb. per cubic foot.

First of all one half of the intrados of the central arch is set out, since it is only necessary to calculate one half of a symmetrically loaded arch, as the stresses in each half are the same.

Set up A B, the thickness of the keystone as determined by Rankine's rule, which may be expressed as follows:—

The depth of the keystone of an arch should be:

for a single arch ... ... $\sqrt{\cdot 12 \times \text{radius}}$,
for an arch of a series ... $\sqrt{\cdot 17 \times \text{radius}}$.

In this case A B $= \sqrt{17 \times 20{\cdot}5}$ feet
$= 1{\cdot}86$ feet.

Complete the arch ring A B D C by striking the arc A C from the centre O and mark off the centre third, J K, L M.

ABUTMENTS should be made of the depth given by Troughtwain's formula—

Where T = thickness of abutments in feet =

$$\frac{\text{radius}}{5} + \frac{\text{rise}}{10} + 2 \text{ ft.}$$

$$\text{In this case } T = \frac{20{\cdot}5}{5} + \frac{6{\cdot}5}{10} + 2$$

$$= 6{\cdot}75 \text{ feet.}$$

Set off D E horizontally equal to 6·75 ft.

BACKING.—The calculations are carried out under the assumptions that the arch is under the influence of a live load, and it is therefore logical to suppose that if the arch is stable when loaded it will be unstable when unloaded, and means must be provided for keeping it in position. For this purpose *backing* is used, and the amount of backing necessary has been found principally by experience. Set up E F vertically equal to $\frac{1}{2}$ H A, and draw F G tangentially to the extrados of the arch. Then E F, F G is the outline of the backing which is composed of masonry in horizontal or concentric courses or sometimes of concrete. Draw the line N P to represent the surface of the road, allowing enough earth above the arch to prevent wearing or vibration in the arch ring at the crown.

The live load assumed in this case is 120 lb. per square foot.

From P set up P Q $= \frac{120}{100}$ ft. $= \frac{\text{wt. of live load per sq. foot}}{\text{wt. of filling per sq. foot}}$;

and draw horizontal line Q R. The space between P N and Q R represents the live load reduced to terms of filling. The whole load above the arch ring is now in terms of filling. Now divide the arch and load up into convenient equal divisions as shown by the thin dash lines.

Make A S $= \frac{100}{160}$ or $\frac{5}{8}$ A Q, T U $= \frac{5}{8}$ T V, W X $= \frac{5}{8}$ W Y, and so on for each divisional line, and join S U X...Z. This represents the whole load in terms of masonry. Bisect the divisions by the lines A′ B′, C′ D′, E′ F′, &c. Then A′ B′, C′ D′, E′ F′, &c., represent the loads of each division in magnitude and direction.

Set these divisions down to scale and find the centre of gravity a, of the first *two* loads by the method shown in Fig. 13. Project a on to the horizontal through J, and call this point β; then try if it is possible to draw a tangent from β to that part of the lower line of the inner third that lies within the *second* division. If it is not possible, find the centre of gravity γ, of the first *three* loads, and from its projection δ on J π try if it is possible to draw a tangent to that part of the lower line of the middle third that lies within the *third* division. Proceed in this manner until a point has been found from which such a tangent can be drawn. If, as in this case, no such point can be obtained, then join the projection π of the centre of gravity ϵ of all the loads to M.

From k draw $k\,o$ parallel to M π and join $b\,o$, $c\,o$, &c. Now starting from J construct the polygon J M π, as shown in Fig. 13. The lower sides of this polygon represent the line of pressure. If this line lies wholly within the centre third, then the arch will be stable, but if not the arch must be thickened until it is possible to draw by the above method a line wholly within the middle third.

When the arch ring has been adjusted so as to comply with these conditions, it can be tested for crushing. Now each of the bars J K′, K′ L′, &c., is under a compressional force equal in magnitude to $a\,o$, $b\,o$, &c., on the vector figure, in which each unit of length represents the weight of 2 cubic feet of masonry, since we are considering a slice of the arch 1 foot thick with

divisions 2 feet wide. It is also clear that since the arch is the same thickness throughout it is only necessary to consider the section where the thrust is greatest, *i.e.*, at the abutments.

Now the area of the section of the ring = 1 ft. × 1·86 ft.
= 1·86 square feet.

And the force brought to bear upon it is shown by *k o*, which is = 51 × 2 cubic feet of masonry;

$$= \frac{51 \times 2 \times 160}{2240} \text{ tons}$$
$$= 7\tfrac{2}{7} \text{ tons.}$$

An average stone will bear 400 tons per square foot, so that the ring is absolutely safe against crushing. Half the smaller arch was worked out in the same way. In this case it is found that it is possible to draw from ϕ, the projection of the centre of gravity of the first six loads on the horizontal line through M, a tangent to that part of the lower line of the middle third which lies within the sixth division.

By drawing these tangents the points are found round which rotation is more likely to take place.

In this smaller arch the line of pressure comes outside the arch ring, and rotation would tend to take place round the point H^1. It is obvious, however, that the backing will prevent such rotation. Theoretically, it is supposed that an arch springs from the point where the bed joint of a voussoir is inclined at the angle of repose of the masonry of which the arch is constructed, so that the lower portions of a semicircular arch may be regarded as the abutments of the upper portion.

The final thrusts, T_1 and T_2 are produced to meet at N^1 and the resultant R_1 found by the parallelogram of forces,

It has been shown that in the stress diagram for the larger arch each unit of length represents the weight of 2 cubic feet of masonry. Now in the smaller arch the arch has been divided up into divisions of 1 foot, so as to produce a more exact line of resistance. It is therefore obvious that each unit of length in its stress diagram represents the weight of 1 cubic foot of masonry. $T_1 = k\,o = 51 \times 2$ cubic feet of masonry $= 102$ cubic feet of masonry, while $T_2 = 36{\cdot}5$ cubic feet of masonry. R_1 works out as 124 cubic feet of masonry. This is combined with W_1, the weight of the figure L′ M′ D K O′ Q′ R′ E J′ K′. The area of this figure is 155 square feet. Hence its weight is that of 155 cubic feet of masonry. The resultant R_2 works out to 275 cubic feet of masonry,

$$\text{which is} = \frac{275 \times 160}{2240} \text{ tons},$$
$$= 19{\cdot}64 \text{ tons}.$$

The abutments have had to be thickened to keep the direction of R_2 within the middle third.

The breadth of the base is 9·5 feet; hence pressure on the soil is $= \frac{19{\cdot}64}{9{\cdot}5} = 2{\cdot}07$ tons per sq. foot.

This will be safely borne by a good soil. If the soil is bad the foundation must be battered or stepped out.

In the same way the end abutment is calculated; the thrust T_3 is combined with W_2, the weight of the abutment, to find R_3. R_3 is combined with the earth pressure P_1, found as shown in Chapter XVIII., acting at a point U′, $\frac{1}{3}$ the height S′ T. Thus the resultant R_4 is found, which must pass everywhere within the middle third of the abutment, for which purpose it has been found necessary to batter out below ground level.

Fig. 123 shows an arch of a somewhat different

character from that shown in Fig. 122. It is built of armoured concrete in ribs and panels as shown in the half bay sections on the right of the figure.

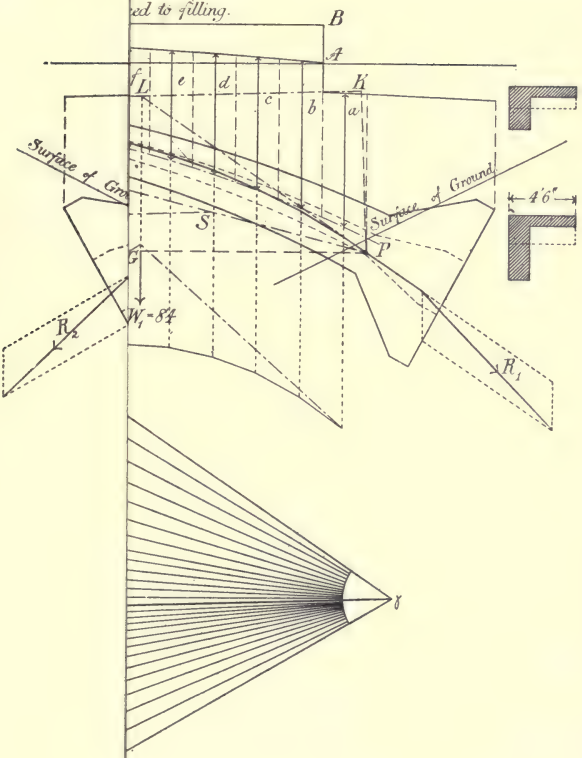
It must have occurred to the reader that an arch is more likely to fail when loaded on one side only, so that when a large gap has to be spanned and heavy live loads carried it is usual to assume a heavy irregular live load on one side only, as indicated by the line A B C D E F.

Both sides must be calculated, and this is done as shown in Fig. 123. In calculating the loads on each division it should be noted that the concrete must be averaged as shown by the dotted lines on the sections. This average is shown on the general drawing by the chain line through the arch ring, and the loads must be measured between the arrow heads.

In this case the important fact to notice is that the thrust at the crown is not horizontal, and its direction must therefore be calculated. To do this we first select the three points O, P and Q, all within the centre third, O being at the crown and P and Q at the springing. Find the centres of gravity of the loads on each half and the moments of the combined loads on each half acting through these centres of gravity about the points P and Q. For stability in the arch these moments must be equal to the moment of the thrust about P and Q. To find this thrust, join O P and divide it at S, so that

$$\begin{aligned} \text{O P : O S} &= \text{moment of } W_1 \text{ about P : moment of } W_2 \text{ about Q} \\ &= 84 \times \text{G P} : 64 = \text{H Q} \\ &\quad 1316 \quad : 960. \end{aligned}$$

Join Q S, and through O draw J O K parallel to Q S. Then J O K represents the direction of the thrust, and its magnitude is equal to either $\frac{84 \times \text{G P}}{\text{K P}}$ or $\frac{64 \times \text{H Q}}{\text{J Q}}$.



[*To face page* 186.

Now set down the vertical loads for the stress diagram and draw $o\,\gamma$ equal and parallel to the thrust as found above. Join L P and M Q, and from α and β in the stress diagram draw lines respectively parallel to L P and M Q. If the drawing and working are accurate they will meet at γ, forming an excellent check upon the working. Join $b\,\gamma$, $c\,\gamma$, $d\,\gamma$, &c., and plot the line of pressure as shown in Fig. 122. If the line of pressure comes outside the middle third, new points O, P and Q must be selected, and another line found. If it is impossible to get a line of pressure within the middle third, the ribs in this case would have to be deepened. Had the line of pressure been started from the point O′, the line of pressure would have come right outside the middle third. It must be borne in mind, therefore, that with unsymmetrically loaded arches it is a matter of experimenting until it is found impossible to obtain the line of pressure necessary for stability, and that experience of arches under different loads will enable a student to pitch upon the correct points (O, P, Q) at the outset of his calculation.

Arches such as occur in the practice of architecture have not been mentioned earlier in this chapter because, as a rule, their thickness has been established by experience, but if it becomes necessary to calculate such arches it is done in exactly the same way as shown in Figs. 122 and 123. Usually much more simple cases are presented, there being no live load to calculate for and the dead load being in terms of masonry or of brickwork in the first place.

CHAPTER XXII.

BUTTRESSES.

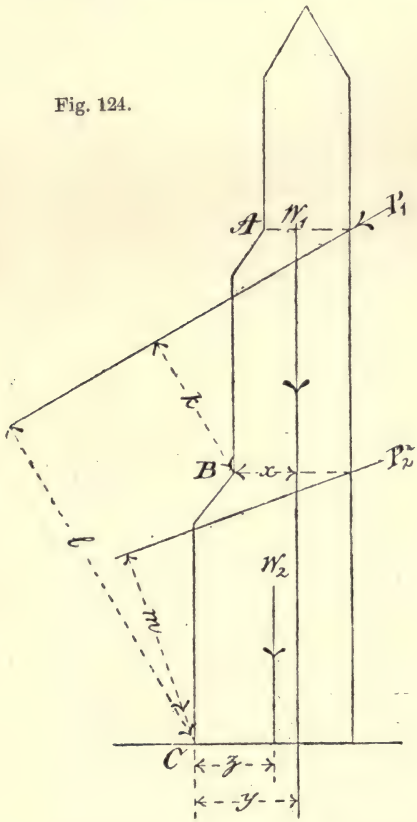
In considering the question of buttresses, it is only necessary to depend upon principles already fully enunciated. The thrusts to which they are subjected once determined, the overturning power which these possess is readily measured by taking moments, just as is done in the case of walls, about their outer edges—and, to obviate overturning, this power or tendency must be at least equated or exceeded by the power of the buttress itself, measured by the moment of its weight about the same outer edge, to resist this overturning. Thus, no special difficulties are presented.

To take one special case, the ordinary one shown in Fig. 124 should be quite sufficient to explain all that is necessary. P_1 is supposed to be the ascertained thrust from an arch flying buttress, and P_2 to be that from the principal of an aisle roof. Of course, at the point A there is no overturning tendency. The weight of the pinnacle above this point (which pinnacle may be as ornate as may be desired so long as it be heavy enough to do its work) is only needed as part of the weight of the buttress above B, acting as W_1 collected at its centre of gravity, with the leverage x, to counterbalance the thrust P_1 acting with the leverage K. Thus, taking moments about B_1—

$W_1 \times x$, must be greater than $P_1 \times k$.

Similarly, supposing W_2 to be the weight of the portion of the buttresses lying between B and C, and to be

Fig. 124.



located at its centre of gravity, then the moment of overturning about the point C becomes—

$$P_1 \times l + P_2 \times m,$$

and the moment of stability about the same point is—

$$W_1 \times y + W_2 \times z.$$

Again, the moment of stability must at least equal, and should exceed, the moment of overturning, and, as explained in an earlier chapter, to secure safety the points B and C, around which moments are to be taken, should be fixed at one-third of the thickness of the buttress from its outer edge.

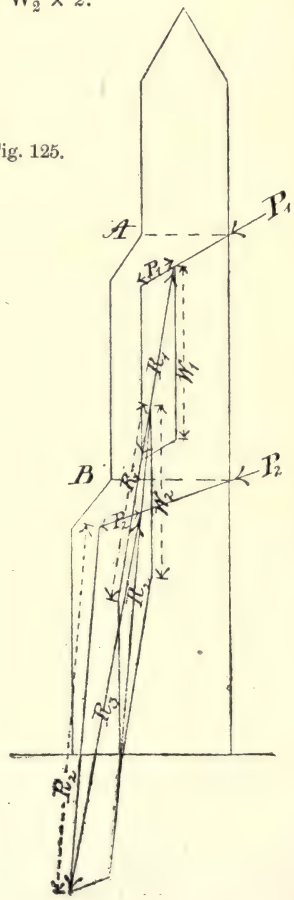


Fig. 125.

It will readily be seen that, by trial, the problem can equally well be solved diagrammatically, as shown in Fig. 125.

P_1 and W_1 are combined where they meet, and their resultant R_1 found. Then this is combined with W_2, and a fresh resultant R_2 found, which is again combined with P_2, and a third resultant R_3

found. Then R_1 must pass within the joint at B, as against overturning, or within its middle third as for safety, and similarly R_3 must pass within the joint at C, as against overturning, and within its middle third for safety.

EXAMINATION PAPERS.

SET AT THE R.I.B.A. INTERMEDIATE EXAMINATION IN JUNE, 1903.

1. Explain fully the meaning of the six words in italics in the following sentences:—A post in *tension* will bear *safely* . . . lbs. to the inch. The *breaking* weight on a beam in *transverse strain* is . . . cwts. The strains on the various members forming a truss are found *graphically*.
2. Why is a king-post truss more secure than a queen-post truss? What precautions do you take against a queen-post truss failing if all the timbers forming the several members are in themselves sufficiently strong for their purpose? Why is a queen-post truss used frequently in preference to a king-post truss?
3. What is the safe weight to put on a fir beam 8 inches wide, 12 inches deep, and 10 feet clear span? The weight assumed in the centre of the beam.
4. What is the equivalent weight at the centre of a beam 16 feet span in clear of the sum of the following three loads? One is 3 tons 3 feet from one end, the second is 5 tons 5 feet from the same end, and the third is 10 tons 6 feet from the other end.

Note.—Apparently the term "equivalent weight" is used to denote that which would produce the same maximum bending moment.—G.A.T.M.

5. A gallery in a hall, projecting 8 feet out from the wall, is carried on fir cantilevers 8 feet long and 10 feet apart:
   (*a*) What is the weight you would assume each cantilever has to carry?
   (*b*) Draw a sketch, about 4 inches wide, of a cantilever (construction only) in the shape of a right-angled triangle, the hypothenuse at an angle of 60° with the brick wall of the building.
   (*c*) Having ascertained the weight on the top member of the cantilever (Question *a*), what scantling would you use, assuming the constant for safe weight distributed in the ordinary formula is $1\frac{1}{4}$ (cwt.)?
   (*d*) What scantling would you make the hypothenuse, and why? The resistance to buckling may be taken in this example at 300 lb. per inch.
   (*e*) The hypothenuse is housed into the vertical post of the cantilever, and this rests on a stone corbel projecting from a brick wall. What weight comes on to the corbel? What area of stone would you put projecting from the wall to take this weight? Draw the corbel full size.

Set at the R.I.B.A. Final Examination, in June, 1903.

1. Draw four different plans of steel stanchions one-fourth full size, and figure the dimensions. Length of stanchion to be assumed 16 feet and weight to be borne 30 tons. State under each plan if such

stanchion is intended for the centre of a building, or a shop front, or against a party wall. For the purpose of simplifying the answer, the metal in the stanchion may be assumed to bear $1\frac{1}{4}$ tons to the inch.

2. What weight will the following steel joist bear distributed with safety? Size of rolled joist 16 inches by 8 inches out of $\frac{3}{4}$-inch metal to flanges with clear span of 20 feet. Draw such a joist full size.
3. If the last named joist had to rest on the head of the previously described stanchion, and had to carry a fireproof floor 8 inches deep in all, the top of the fireproof floor level with the top of the joist, show sectional construction half full size.
4. Sketch a section through a cast-iron beam or girder, and state your reason why the top and bottom flanges are the same or different sizes.
5. A heavy building rests partly on a brick pier 6 feet by 3 feet, calculated to take safely 6 tons to the foot. It is decided to replace this by an iron column. Sketch the base and spreading under column which has to be erected on the old concrete under the brick footings of the pier, which has to be left undisturbed for certain reasons. Assume the ordinary spreading footings with 9 feet by 6 feet of concrete under brickwork.

SET AT THE R.I.B.A. INTERMEDIATE EXAMINATION IN JUNE, 1902.

1. What is the safe weight to put on two 11 inch by 3 inch deals bolted together, having a clear span of 12 feet, weight distributed?
2. What would the deals in the previous question carry

safely if the weight were all put at 3 feet from one end of the deals?

3. Should the deals in the previous questions prove too weak, how would you stiffen or strengthen them in a warehouse without appreciably disturbing the flooring?
4. Supposing a cast-iron or steel + stanchion can carry safely $1\frac{1}{2}$ ton to the inch of metal, what section is necessary for a weight of 36 tons? Give horizontal section of stanchion to scale.
5. A column half-way between two external walls has to support the middle of a girder 28 feet long. It is 10 feet away from the adjoining column on either side. The girder supports a floor weighing 42 lb. to the foot, and the weight to be carried by the floor is $1\frac{1}{2}$ cwt. to the foot. What weight has the column to carry?
6. A weight of 60 tons has to be carried on four similar brick piers, $2\frac{1}{2}$ bricks thick, supporting three arches, the weight distributed equally over the arches. Assuming the brickwork will carry 5 tons per foot safely, give a plan to scale of one of the piers.
7. Give the reciprocal diagram of a king-post truss, showing step by step how you make it, and explain clearly what purpose the diagram serves.

Set at the R.I.B.A. Final Examination, in July, 1902.

Note.—The questions omitted did not deal with "Stresses and Thrusts."

1. A pair of rolled-steel joists carry in the centre of their bearing a column which in its turn carries

the upper part of a building. The joists rest on a pair of stanchions 16 feet apart in the clear. The column and superincumbent weight are to be taken at 24 tons.

(*a*) Calculate the scantling of the joists.

(*b*) Sketch the junction of the column and the joists, giving perspective view, or plan and cross section.

2. What difference do you make in the setting out of the joists as in Question 1, if, instead of a column, the joists carry a wall 14 inches, 18 inches, or 21 inches, thick? Give sections.

3. Draw full size the horizontal section of a cast-iron or wrought-iron or steel stanchion 18 feet long, carrying one end of the joists in Question 1, and a direct load of 6 tons in addition. Figure the dimensions as well as drawing them. Draw the head of the stanchion full size. The candidate must state if he has chosen steel, wrought iron, or cast iron.

6. (*a*) What weight do you reckon per foot super for the floor of a private house?

(*b*) What is the weight of brick walling per foot cube?

(*c*) What is the safe tensile strain of a wrought-iron bar 1 inch square?

(*d*) What is the safe compressive strength of cast iron?

(*e*) What is the weight of a gallon of water?

Set at the R.I.B.A. Intermediate Examination in June, 1901.

1. Give the safe load per foot superficial you would place on foundation on

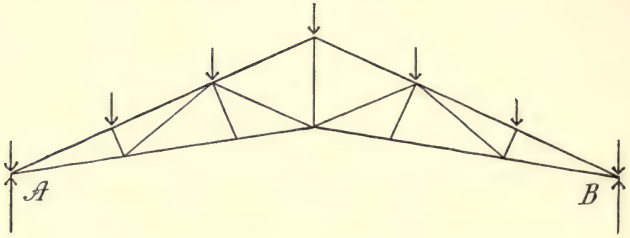
(*a*) London clay;

(*b*) Ordinary ballast.

2. What load would you put on a pier 12 feet high, 2 feet square, built of
   (*a*) Stock brickwork in cement?
   (*b*) Stock brickwork in lime mortar?
3. Give the size of an ordinary fir beam, 12 feet bearing in the clear, to carry with safety 144 superficial feet of floor (weight distributed)
   (*a*) In a heavy warehouse;
   (*b*) In an ordinary dwelling-house.
4. Give the ordinary formula for calculating the safe load to be put on an iron or steel girder, and give the meaning of the different letters in the formula and the constant
   (*a*) For wrought iron;
   (*b*) For steel.
5. What is the weight of a cubic foot of water?
   How much water in gallons is there in a cubic foot?
6. What is the pressure per foot superficial to be allowed for the wind on a roof of 45° pitch?

SET AT AN EXAMINATION FOR ASSISTANT CIVIL ENGINEERS, ADMIRALTY, IN JULY, 1901.

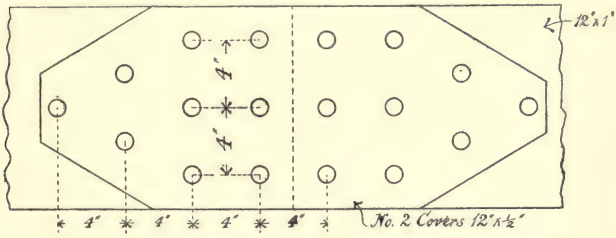
1. Define the following terms and give an illustration of each:—
   Resilience.
   Moment of Resistance.
   Stress.
   Strain.
2. Draw the stress diagram for the frame shown on next page, assuming the joints to be loaded with equal weights in the direction of the arrows. The arrows A and B indicate the directions of the reactions.



3. A number of braced girders of similar dimensions are designed to carry an evenly-distributed load of, say, 100 tons, and are required to be tested with full load; for convenience of testing, the contractor proposed to place two girders back to back, with a hydraulic jack between them at the centre, and to apply a load of 50 tons. Is this a fair test? Give your reasons.
4. Draw the diagram of the forces keeping a retaining wall 12 feet high in equilibrium. Assume the wall to weigh 180 lb. per cubic foot, and the earth backing 120 lb. per cubic foot, and to have an angle of repose of 35°.
5. State Gordon's formula for the strength of columns, and ascertain the breaking weight of a solid cast-iron column 10 ft. long and 4 ins. diameter; $f = 80{,}300$ lb. per square inch; $a = \frac{1}{400}$.
6. A beam 40 ft. long is loaded with three weights of 5, 15, and 10 tons placed 10 ft. apart, the 15 ton weight being at the centre of the span. Draw the diagram of the bending moment and the shearing stress.

Set in an Examination for Assistant Civil Engineers, Admiralty, in July, 1898.

1. Define the terms—(*a*) bending moment, (*b*) moment of resistance, (*c*) moment of inertia, (*d*) modulus of elasticity, and give examples of each.
2. State Gordon's formula for columns, and find the breaking load of a cast-iron column 15 ft. long, 9 inches external diameter, $\frac{3}{4}$-inch metal and with flat ends, taking $a = \frac{1}{800}$ and $f = 80{,}000$.
3. Draw the stress diagram of the bending moment, and shearing forces of a plate girder 50 ft. clear span, when submitted to a uniform dead load of $1\frac{1}{4}$ tons per foot run, and a live or travelling load of 5 tons when at the centre and at a point 12 ft. 6 ins. from the end.
4. Sketch the cross-section of the above girder when 5 ft. deep, taking the unit stresses as 5 tons and 4 tons per square inch for tension and compression respectively, the net area being taken for tension.
5. If the joint shown in the annexed figure were tested to destruction with a tensile strain, where would it fail, and why?



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